EXPERIMENTAL STUDIES OF THE BEHAVIOR OF SPAR TYPE STABLE PLATFORMS IN WAVES

by

Bruce H. Adee

Kwang June Bai

This research was carried out under the Naval Ship Systems Command General Hydromechanics Research Program administered by the Naval Ship Research and Development Center

and

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ERRATUM

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The following was omitted from the published report, and this sheet should be inserted, as page 68a:

THE WRITE-UP OF THE SPAR PROGRAM

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NO.  DATA AND FORMAT

 1   TITLE
     FORMAT(10AB)
     1 CARD

 2   NOSEC,G,ROE,W,ZGB
     NUMBER OF SECTIONS,ACCELERATION OF GRAVITY(FT/SEC/SEC),FLUID
     DENSITY(SLUGS/(FT)**4), WEIGHT(LB),C.G. ABOVE BASE (FT)
     FORMAT(I10,4F10.4)
     1 CARD

 3   NOSECT(I),NTYPE(I),AZ(I+1),A(I),C(I),D(I)
     SECTION NUMBER,SECTION TYPE, TOP OF SECTION ABOVE BASE(FT),ETC.
     DEPENDING ON SECTION TYPE
     FORMAT(2I10,5F10.4)
     NOSEC CARDS, ONE FOR EACH SECTION

 4   K
     THE NUMBER OF WAVE NUMBERS
     FORMAT (I10)
     1 CARD
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READ BEFORE PAGE 69

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ABSTRACT

Newman has developed a linearized theory for the motions of a slender body of revolution, with vertical axis, which is floating in the presence of regular waves. In the present paper a series of experimental investigations were made and compared with Newman's theory. Experimental measurements of motions were made in regular and irregular long crested waves. Pressures at several locations on the models were also measured and compared with the theory. The measurements of motions give excellent agreement with theory for slender body. An extended formula was developed for heave motion for small slenderness ratio of the body. The theoretical prediction for pressure on the body also was found to give excellent agreement with the experimental measurement except near the free surface. Observation of vortex generation was made by electrolysis.

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NOMENCLATURE

A		incident wave amplitude
f		frequency of oscillations (cps)
g		acceleration of gravity
I		moment of inertia of buoy in pitch about the center of gravity
H		buoy draft
K		wave number ω^2/g
k_Y		radius of gyration , $\sqrt{I/m}$
m		buoy mass
m'		added mass of buoy in heave motion
p		pressure
$\bar{p}$		$p/\rho g A$
P_n	$=$	$\frac{\rho}{m}\int_{-H}^{0}(z-z_G)^n\, S(z)\, dz \quad (n=1,2)$
Q_n	$=$	$\frac{\rho}{m}\int_{-H}^{0}(z-z_G)^n\, S(z)\, e^{kz}\, dz \quad (n=0,1)$
$S(z)$		Section area of buoy
s	$=$	$\frac{m'}{m}$
x, y, z		cartesian coordinate system
z_G		position of center of gravity of buoy
χ		vertical prismatic coefficient
ρ		density of fluid
ϕ		velocity potential
ζ		heave displacement of buoy
ξ		surge displacement of buoy
ψ		pitch angle of buoy
ω		circular frequency of oscillations
T_0		natural period (sec.)

$\varkappa$	damping coefficient, for example, $\ddot{y}(t) + 2\varkappa\dot{y} + ky = 0$
$\bar{\varkappa}$	non-dimensional damping coefficient
ω_0	natural angular frequency
$(kH)_0$	the KH value for natural frequency

I. INTRODUCTION

The inherent stability of slender poles floating with the long axis upright has long been recognized. Utilization of this principle has ranged from simple wave poles to more complex oil drilling platforms. The concept has also been applied in the design of stable, floating platforms, called "spar buoys," employed in oceanographic research.

The construction of a spar buoy was proposed by Fisher and Spiess and reached fruition with the delivery of FLIP[3] (Floating Instrument Platform) to the Scripps Institution in 1962. The original intent was to provide a stable platform from which mid-ocean acoustical experiments could be performed. Of course, the scope of potential investigations in the field of oceanography which require a stable platform is very broad, the utilization of "spar buoys" for various experiments should continue to increase in the future.

Besides FLIP, there are three other "spar buoys" in existence. One is called SPAR[4] (Seagoing Platform for Acoustical Research) and is operated by the Navy in the Atlantic. Another "spar buoy" is operated by the Musee Oceanographique in the Mediterranean. The most recent addition to the growing fleet of "spar buoys" is called POP[5] (Perpendicular Ocean Platform) which is operated by General Motors from Santa Barbara, California.

The interest generated by FLIP has led a few investigators to develop theories for the prediction of the forces acting on the buoy in the upright direction and the motions excited by waves. These theories, when practically applied, have great value not only from the standpoint of design of future spar buoys, but also for utilization in oceanographic experiments when it is necessary to remove errors introduced into collected data by buoy motions.

In 1963, Newman[1], published his linearized theory on the motions of "spar buoys." Newman's paper served as the stimulus for the present investigation. It was used in the development of theoretical predictions for the pressures acting on a buoy in waves and for predicting the motions of a buoy excited by waves. Another method for the prediction of "spar buoy" motions was proposed by Rudnick[2] and may be viewed as a simplified approach to the problem that leads to essentially the same predicted values of the motions as Newman's theory.

The goals of the present investigation are divided into two major categories; theoretical and experimental. The theoretical portion included a search for methods of predicting the exciting forces and motions of a "spar buoy" in waves. After this was completed, a method, based on the use of a digital computer, was developed for applving these theories to a "spar buoy" of arbitrary dimensions.

The experimental phase of the work was then to evaluate the merits of the theories. For this purpose a series of models of various slenderness ratios (draft/radius) and bottom configurations (flat or conical bottom) was constructed. This permitted a careful appraisal of the limits of applicability of the linearized assumptions made by Newman. It was found that the agreement between theory and experiment was excellent for slenderness ratios greater than about seventeen. However, as expected, a modification of the theory accounting for added mass in heave was necessary for slenderness ratios lower than seventeen.

II. THEORETICAL BACKGROUND AND COMPUTER PROGRAM

1. Newman's Theory

Newman approaches the problem on the basis of the classical, inviscid motion theory. He seeks a velocity potential, $\phi(x, y, z, t)$ which satisfies Laplace's equation subject to the following boundary conditions:

1. The kinematic boundary condition on the body.
2. The free surface boundary condition.
3. The radiation condition.

In deriving the hydrodynamic forces and moments acting on the body, it is assumed that the incident waves and the oscillations of the body are small, and the body is slender. The analysis with only first order terms in the body's diameter leads to undamped resonance oscillations of infinite amplitude. To analyze motions near resonance, it is necessary to introduce damping forces which are of second order with respect to the diameter-length ratio.

Adopting the nomenclature of Newman's report(1), the amplitude of waves, heave, pitch, and surge will be described by A, ζ, ψ, ξ respectively (Figure 1 portrays the coordinate system) Denoting:

$$I = m K_Y^2$$

$$\chi = \frac{m}{\rho H S(0)}$$

$$P_n = \frac{\rho}{m} \int_{-H}^{0} (z - z_G)^n S(z)\, dz \quad (n = 1, 2)$$

$$Q_n(K) = \frac{\rho}{m} \int_{-H}^{0} e^{Kz} (z - z_G)^n S(z)\, dz \quad (n = 0, 1)$$

Solutions of Newman's undamped equations of motion are:

$$\zeta = A\left[\frac{1-\chi KHQ_0}{1-\chi KH}\right]\sin\omega t \tag{1}$$

$$\xi = 2A\left[\frac{P_1Q_1 - Q_0(P_2 + K_Y^2 - P_1/K)}{2(P_2 + K_Y^2 - P_1/K) - P_1^2}\right]\cos\omega t \tag{2}$$

$$\psi = 2A\left[\frac{P_1Q_0 - 2Q_1}{2(P_2 + K_Y^2 - P_1/K) - P_1^2}\right]\cos\omega t \tag{3}$$

From these equations it is seen that resonance occurs in heave when

$$K = \frac{1}{\chi H}$$

in pitch and surge when

$$K = \frac{P_1}{P_2 + K_Y^2 - \frac{1}{2}P_1^2}$$

Newman proceeds with his analysis to compute a damping term and includes it in a set of damped equations of motion. The solution of this set of equations and the solution of the equations of the extension of Newman's theory to bodies of small slenderness ratio [1] have been obtained and included in the computer program for the evaluation of the motions.

The solutions of the damped equations of motion are extremely cumbersome and are not of practical importance because computational experience has shown them to be unnecessary for the configurations examined except in a very narrow region near the resonant frequency.

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[1] defined as the ratio of draft to mean radius of the body.

Solutions of Newman's damped equations of motion are:

$$\zeta = \frac{A(1-\chi KHQ_0)}{\{(1-\chi KH)^2 + [\frac{1}{2}\frac{mK^2}{\rho\chi H}(1-\chi KHQ_0)^2]^2\}^{\frac{1}{2}}} \sin(\omega t+\alpha) \quad (4)$$

$$\xi = \frac{d_3}{(d_1^2+\omega^2 d_2^2)^{\frac{1}{2}}} \sin(\omega t+\beta) \quad (5)$$

$$\psi = \frac{b_3}{(b_2^2+\omega^2 b_1^2)^{\frac{1}{2}}} \sin(\omega t+\gamma) \quad (6)$$

where

$$b_1 = \frac{1}{2}\frac{m}{\omega\rho}K^3[-2Q_0Q_1P_1 + (P_2 - P_1/K + K_\gamma^2)Q_0^2 + 2Q_1^2]$$

$$b_2 = P_1^2 - 2P_2 + 2P_1/K - 2K_\gamma^2$$

$$b_3 = 2A(-P_1Q_0 + 2Q_1)$$

$$d_1 = 2(P_2 - P_1/K + K_\gamma^2) - P_1^2$$

$$d_2 = \frac{m}{2\omega\rho}K^3[2P_1Q_0Q_1 - Q_0^2(P_2 - P_1/K + K_\gamma^2) - 2Q_1^2]$$

$$d_3 = 2A[P_1Q_1 - Q_0(P_2 - P_1/K + K_\gamma^2)]$$

$$\alpha = \tan^{-1}\left\{-\frac{mK^2(1-\chi KHQ_0)^2}{2\rho\chi H(1-\chi KH)}\right\}$$

$$\beta = \tan^{-1}\left(\frac{d_1}{d_2\omega}\right)$$

$$\gamma = \tan^{-1}\left(\frac{b_2}{b_1\omega}\right)$$

It should be noted that this theory as well as the presentation of Rudnick's theory[2] described later in this chapter deal with bodies of revolution which move only in one plane (i.e., only three degree of freedom; heave, pitch, surge).

2. Extension of Newman's Theory to Bodies of Small Slenderness Ratio

The formula in the preceding section do not include heave added mass. This is a consequence of utilizing the slender body theory in the solution of the potential problem. As the model deviates from the slender body assumption made in Newman's theory, a modification is necessary.

In order to extend Newman's theory to include cases of small slenderness ratio added mass and viscous damping terms should be included in the equation of motion. The primary effect of the heave added mass term is to shift the resonant frequency, while viscous damping tends to decrease the amplitude of the response of the motion (i.e., to change the magnification factor).

Since the damping is small in either case, the effect on body motions in a realistic seaway will be more strongly influenced by changes in the natural frequency. Therefore, the viscous damping was not taken into account. As a reference, the measured total damping, which is the sum of the viscous damping and the damping due to the energy dissipation through waves, is given in Table 2 in non-dimensional form.

In order to incorporate heave added mass into the equations of motion, we first define two new added mass coefficients as follows:

$$S = \frac{m'}{m}$$

$$\gamma = \frac{m}{m+m'} = \frac{1}{1+S}$$

It is well know that the added mass coefficient is a function of not only the geometry of the body but also of the frequency of the motion. In order to find an approximate value of this added mass coefficient as a function of the slenderness ratio, free oscillation experiments were made with several models representing a considerable range of slenderness ratio. The results of these experiments are plotted in Figure 5.

The modified heave motion including the above heave added mass is

$$\zeta = \frac{\frac{g}{g} A\,(1-\chi K H Q_0)}{\left\{\left(\frac{g}{g}-\chi K H\right)^2 + \left[\frac{1}{2}\,\frac{g}{g}\,\frac{m K^2}{\rho \chi H}\,(1-\chi K H Q_0)^2\right]^2\right\}^{\frac{1}{2}}} \sin(\omega t - \alpha') \qquad (7)$$

where

$$\alpha' = \tan^{-1}\left\{\frac{\frac{g}{g}\, m^2 \omega^2 K\,(1-\chi K H Q_0)}{2 \rho \chi^2 H^2 \left(\frac{g}{g}\,\rho g\, S(\omega) - m\omega^2\right)}\right\}$$

3. Rudnick's Theory

Rudnick approaches the problem by considering the three major contributors to the total force system acting on the buoy. These are the forces associated with the mass of the buoy, the hydrodynamic accelerations, and hydrodynamic response to transverse accelerations. Summation of the forces and moments yields one vector equation for forces and one for moments. Taking the components and performing the requisite integrations leads to the equations of motion. For purposes of comparison of the two methods, Rudnick's solutions are presented here in terms of the same set of variables as used in Newman's theory. The solutions are:

$$\zeta = A\left[\frac{1-\chi K H Q_0}{1-\chi K H}\right] \sin \omega t \tag{7}$$

$$\xi = A Q_0 \cos \omega t \tag{8}$$

$$\psi = A\left(\frac{2Q_1 - P_1 Q_0}{P_2 - K_Y^2 - P_1/K}\right) \cos \omega t \tag{9}$$

Resonance in heave occurs when:

$$K = \frac{1}{\chi H}$$

in pitch when:

$$K = \frac{P_1}{P_2 + K_Y^2}$$

and there is no surge resonance.

4. Computer Program

A digital computer program was written in the FORTRAN IV language for evaluating the theoretical predictions of buoy motions. A listing is given in Appendix

The integrations involved in evaluating the coefficients P_1, P_2, Q_0, and Q_1 may be performed easily for several possible shapes of the buoy (i.e., cylinderical, conical). These integrations may then be stored in subprograms in the form of an equation for the indefinite integral (one subroutine for each possible shape).

Any possible underwater configuration of a buoy may then be broken into a series of cylindrical and conical segments. The sementation of the buoy is read into the main program along with the other data on the density of the water, the weight of the buoy, and its dimensions. The main program then calls on the subprograms to evaluate P_1, P_2, Q_0, Q_1 for each segment of the buoy and sums the results from each subprogram to obtain the overall values for the buoy. Once these values are found, the evaluation of equations (1) through (9) for the motions is a simple matter. The actual operation of the program is depicted in a flow chart shown schematically in Figure 32.

III. EXPERIMENTAL TECHNIQUE

The construction of models and experimental techniques are described in this chapter. Pitch, heave, and surge pressures and the vortex generation around the model were observed or measured. Four methods were used in different experiments to measure the model motion. Pressure gages were introduced to measure the pressure on the model and electrolysis was introduced in order to visualize the flow around the model. They are explained in detail in each section.

1. Models

Four models of different diameters were made of aluminum pipe. Each has two bottom attachments. One is a flat disk and the other is a circular cone shape with a fixed height of three inches to fit into the pipe. Therefore the family incorporates systematic variation in transverse dimensions only.

The construction of models is illustrated in Figure 2 and their dimensions and characteristics in Table 1. The measurements performed in each case is also described in the table.

2. Multiple Flash Photograph Technique

This method employs a camera used for taking still photographs. The camera was mounted on a set of rails about four feet long and placed parallel to the towing tank so that a picture could be taken through the glass panel at the side of the tank. A black background cloth was placed on the opposite side of the tank, and the model was also painted black with white identification stripes. During the test the entire tank area was from the stroboscopic light directed at the model and set to flash at intervals of one-tenth of a second.

A test would consist of the following steps:

1. The model was positioned and waves generated.
2. All ligh- were shut off and the stroboscopic light was switchea on.
3. The shutter of the camera was opened and it was slid along the rails.
4. When the camera had moved past one panel of the window the shutter was closed (about 2-3 seconds).

The film used was a polaroid film which produces both a positive and a negative. The positive could be viewed immediately and the results either accepted or rejected after each test. If the results were satisfactory a large scale print was made from the negative.

The motion measurements obtained by using this technique were found to have a great deal of scatter. In addition the limited exposure time permitted only about one and one-half periods of the motion to be recorded. It was, therefore difficult to obtain a dependable average of the amplitude of motion and, consequently, the method was discarded.

3. Motion Transducer Technique

In order to facilitate the evaluation of results as well as to record several cycles of motion, it was felt that an electrical measuring device with its output led to a chart recorder would be desirable. Data gathered by this technique could be viewed immediately and would provide a record over many periods of motion. In this was extraneous motions could be eliminated leaving only "steady state" values.

A "constant thrust" gravity dynamometer was available in the laboratory and had previously been used for towing ship models in head seas. This device was too heavy for the needs of the present experiments, however, so a new design was prepared and

constructed attempting to minimize both weight the friction of moving parts. Figure 4 is a schematic drawing of the motion transducer. A description of this instrument is as follows.

A set of rails was fixed in the direction of propagation of the waves. A light weight subcarriage was mounted on the rails and connected by a wire and pulley to a potentiometer so as to sense surge translations of the model. Four ball bushings were mounted in the subcarriage holding two vertical guide rods which, in turn, are attached to the buoy model through a pitch pivot at their lower end. Wire, a pulley, and a potentiometer system is incorporated between the vertical rods and the subcarriage to measure the heave motion. The pitch motion is sensed by a potentiometer gear driven from the pitch pivot.

There are two drawbacks in this device which limited its successful application. They both arise from the fact that the models are long and slender, thus, the mechanical connection to the model must be at the top. When the model rotates about its center of gravity both rotation about the pitch pivot and surge translation of this point, therefore of the subcarriage result. The friction in the subcarriage therefore introduces a great deal of external pitch damping and at the same time the mass of the subcarriage increases the effective moment of inertia of the system. A correction for these effects is shown in Figure 7.

The second detrimental effect is that tangential motion at pin due to pitch motions must be subtracted from the recorded horizontal translations in order to obtain the true surge at the center of gravity of the model. Since these terms are often of similar magnitude the errors introduced into the measured values of true surge may represent a substantial percentage.

4. Motion Picture Technique

Experimentally this is the simplest technique employed in this work. A 16 MM movie camera is set to shoot through the glass panel in the tank wall. A clock and vertical reference line are placed in the center of the tank so that they will be within each frame. The model is left free and positioned so that it will not drift out of the frame of the picture during the run. For length measurement a reference scale is marked on the model thus alleviating parallax error.

The advantage of this method is that the model remains unencumbered with any device capable of disguising the actual motions. The major disadvantage lies in evaluating the results. To say the least it is a time consuming and most arduous duty. Nevertheless, it remains one of the most accurate and successful techniques of measuring the true buoy motions. It was, therefore, used as a "standard" for evaluating the accuracy of other methods.

5. Accelerometers and Rate Gyro Technique

A. Instruments and Experimental Procedure

In the last two models (i.e., Model No. 3.4) two seismic accelerometers oriented to sense heave and surge motions and one rate gyro-scope were installed. To facilitate installation of these instruments, they were packaged in an instrument module. This module was then placed in the models.

These instruments are:

Accelerometers;

(for heave) - Statham Lab. Model No. f-2-350,
11 V max ± 2G

(for surge) - Statham Lab. Model No. c-1-350,
9 V max ± 1G

Rate Gyro ;

(for pitch) - U.S. TIME Model No. 40, 07-90023,
40 V. Max. Out at 40 Deg/sec.

In this experiment the model was restrained only to prevent it from rotating about its vertical axis of revolution. This was necessary in order to maintain orientation of the pitch gyro and surge accelerometer. Two light aluminum struts were installed to project upwards on top of the model. The model was positioned in the tank so that these struts were restained between two horizontal, parallel stainless steel rods mounted longitudinally on the carriage in the center of the towing tank. These restraining rods were well lubricated before each experiment. The instrument leads from the measuring instruments in the model were made from light weight phonograph arm wire and were loosely hung on the carriage to permit the model to move freely in the direction of propagation of the waves.

It was initially attempted to double integrate the acceleration signals to get the corresponding displacements of motion by using series operational amplifiers connected as integrators. The final value was found to contain large errors as a result of drift and noise in the integrators, so this method was given up. It was however found satisfactory to integrate the angular velocity output from the rate gyro to obtain the pitch displacement signal. The accelerations - the signals of the accelerometers - and the angular displacement obtained by integrating the gyroscope output as well as the wave amplitude were recorded on a strip chart recorder.

Instead of twice integrating the heave and surge accelerations the motion was assumed to be sinusoidal. In this case, acceleration is related to displacement by the following relationship,

$$\ddot{\zeta}(t) = -\omega^2 \zeta(t) \quad , \qquad \zeta(t) = -\frac{1}{\omega^2}\ddot{\zeta}(t)$$

In each experiment the average of the frequency of the acceleration was obtained through several measurements of accelerations recorded on the chart paper. It is obviously not valid to use the above relation when the motion is no longer sinusoidal. However, it is also observed, as we expected from the standpoint of the shallow water theory, that the longer the waves the more the deviation from the sinusoidal waves. It was not considered important to investigate the responses of the motion in long waves of which the wave length is greater than fifty feet since this kind of wave is beyond the expected range of large model motion. For shorter waves the motion was found to be nearly sinusoidal and this relationship could be used.

In order to obtain the pure (true) heave from the measured heave motion a correction should be applied for the effect of pitch. However, since this correction varies as one minus the cosine squared of the pitch angle it may usually be neglected. In the present case the maximum value of the effect of the pitch on heave was only about 3 to 4 per cent of the measured heave motion. On the other hand, the effect of the pitch on the surge acceleration measured by the surge accelerometer was about 90 per cent of the measured value. This was felt to completely obscure the true surge motion and consequently, surge was not evaluated by this technique.

B. Irregular Waves

Irregular waves are generated in the tank by means of a hydraulically driven wave maker in which the length of stroke of the wave maker paddle is controlled by a servo-mechanism in response to a signal recorded on magnetic tape. Using a stock input tape, the time scale may be adjusted by adjusting the speed of the tape recorder and the amplitude of the stroke by adjusting the gain of the servoamplifier.

The random waves were sensed by a resistance type wave probe and recorded in both digital form on magnetic tape and in analogue form on a strip chart recorder. The model motions were recorded in similar fashion.

The digital recorder has the capability of sampling up to sixteen input channels at a rate of one hundred samples per second. In our experiment a total of five channels were used including one empty channel to record angular displacement of pitch motion, heave acceleration, surge acceleration, incident wave aplitude, and the fifth channel was empty as a marker.

Only one model condition, the third model with conical bottom, was chosen for experiments in irregular waves. The wave spectra used were chosen such that their characteristic frequencies were close to the natural frequency of the model.

The spectra of the random waves are shown in Figure 15.

In regular waves the effect of the pitch motion on the heave accelerometer was neglected as mentioned in the preceding subsection, but this effect can not be neglected in the experiment among irregular waves. For the pitch motion is no longer harmonic sinusoidal and the effect of the pitch motion on the heave accelerometer is not negligible. This

correction could be done similarly by the transformation of the coordinate system.

Once the input (wave) and output (true motion response) are obtained the transfer function of the system of the motion can be found. As output, the true heave acceleration and angular displacement of pitch motion were used, and as input the wave. A standard spectral analysis computer program was used for this purpose. In order to compare with this experimental result in irregular waves, the extended form of Newman's theory was used as for a theoretical prediction of the response amplitude operator.

C. Calibration and Wave Measuring Device

To calibrate the accelerometers and integrated rate gyro output, a static method of calibration was applied to the instrument module by displacing it through a known angle. The angular output was thus calibrated directly and the apparent acceleration sensed by the accelerometers was gravity times the sine or cosine of the angle of inclination.

The waves were measured by an electric resistance wave probe. This device consists of a probe passing through the water surface and a bare ground wire on the tank bottom. The electrical resistance between the probe and ground is found to be very linearly with the wetted length of the probe. A Wheatstone bridge is used to detect this resistance change.

6. Pressure Measurement

To measure the pressure samples on the model two pressure gauges were installed on the model surface of the third model with the cone shaped bottom after the model as illustrated in Figure 3. One gauge is located near the water surface and the other is located near the bottom of the cylindrical part of the model.

The two pressure gages used are as follows:

The upper pressure gage;	Statham Model No. PM 222 TC
	Sensitivity 500 MV/V/psi
The lower pressure gage;	Kulite Model No. CPL-125-10
	Sensitivity 1.2 MV/V/psi

To calibrate the pressure gages, a known static water head was applied by moving the model with gauges installed up or down a known distance in the water.

7. Flow Visualization - Vortex Observation

It is of a considerable interest to investigate the effect of viscosity in the motion of the spar buoy in the waves. In the theory, the fluid is assumed inviscid, but the effect of viscosity of water should be taken into account when the motion is large or the slope of the body of the revolution is very large, e.g., near the edge of the flat bottom. It was attempted to observe visually the onset vortex generation near the flat bottom of a model by introducing into this region hydrogen bubbles generated by electrolysis. A fine wire was put on the model and electrically isolated from it. Another electrode was made of copper plate and fixed near the moving model. A D.C. voltage was applied to these electrodes and the resulting electrical current was adjusted

until the size and quantity of the bubbles were suitable for observation.

Two different models were employed to do this experiment, one had a flat disk bottom and the other had a cone shaped bottom. The vortex generation was not observed in the latter, but near the edge of the flat disk bottom, a slight vortex generation was observed due to heave and large pitch motion.

IV. EXPERIMENTAL RESULTS

1. Experimental Results in Regular Waves

The results of the motion picture and motion transducer measurements of the experiments performed with the Model No. 2 with the cone shaped bottom and slenderness ratio of about 17 are presented in Figure 6, 7, 8.

The results of the accelerometer and rate gyro measurements for Models No. 3 and 4 having slenderness ratios of about 10 and 5 for regular waves are presented in Figure 9-14.

The added mass coefficient versus slenderness ratio for heave motion is given in Figure 5.

2. Experiments in Irregular Waves

The wave spectra are presented in Figure 15. The results of the heave and pitch measurements in irregular waves are presented as heave acceleration spectra and pitch angular displacement spectra. The transfer functions for these motions were also obtained and theoretical prediction of those transfer function were computed as previously noted. All of these results are presented in Figure 17 to 29.

3. Pressure Measurement

Pressure measurements for two different depths and three different angular orientations are given in Figure 30 and 31 together with their theoretical predictions from Newman's theory.

V. DISCUSSIONS

1. Motion Picture and Motion Transducer Measurements

The results of motion picture and motion transducer measurements of the experiments performed with Model No. 2 with cone shaped bottom presented in Figure 6, 7, and 8 shows following features.

Heave. Both techniques of measurements show excellent correlation with the theoretical predictions. The scatter of points at higher KH values is understandable in light of the experimental error in measuring these very small motions and the possibility of introducing at random exciting forces that vary from the regular wave pattern.

Pitch. The motion picture results show excellent agreement with theory. However the transducer results are disappointing and illustrate the damping which is introduced by the instrumentation. The reason that damping influences the pitch motion while it does affect the heave can be seen if the system is closely examined. As mentioned previously, pitch friction results from both a rotation about a pivot and surge translation along the rails, while heave friction arises from the four ball bushings and the pulley potentiometer connection.

Surge. Again both techniques yield similar results. It should be noted that a correction should be applied to the potentiometer results for surge in the case of Newman's theory. The effect is most pronounced in the area of predicted surge resonance. It reduces the value KH for resonance slightly and increases the value at higher KH values. The correction is generally small and for the sake of clarity was not plotted. The data does indicate the possibility of a resonance in surge, although much more data is required before a conclusion on this point may be reached.

2. Accelerometer and Rate Gyro Measurements

A. In Regular Waves

The extended formula of Newman's theory for heave gives an excellent prediction for the model of small slenderness ratios.

The measured pitch motion and its theoretical prediction show good agreement with each other for the models of slenderness ratios of 10 and 5.

B. Irregular Waves

The transfer functions of the three different experimental results for the same model condition show excellent agreement each other.

The theoretical prediction and experimental values of the transfer function for the heave acceleration and pitch angular displacement does not give good agreement quantitatively but they give good agreement qualitatively.

3. Pressure Measurement

The pressure measurements at the lower pressure gage near the bottom gives a very good agreement with the calculation of the Newman's theory in all the three different orientations. The upper gauge, however, shows much deviation from the prediction in all the three orientations. It is suggested that this discrepancy results mainly from the effect of the free surface, i.e. we linearized this free surface boundary condition.

4. Applicability of Newman's Theory

For the model of which the slenderness ratio is about 17 Newman's theory gives very good prediction for the heave, pitch, and surge. For the slenderness of 10 and 5, Newman's theory gives a good prediction for pitch, but for heave the extended formula of Newman's theory gives better agreement with the experimental results.

ACKNOWLEDGEMENT

The authors wish to express their appreciation to all those who assisted in preparing this report. Special thanks should go to Professor Paulling for his guidance during the course of this study. Also we wish to thank Professor Wehausen for several consultations and suggestions. During the experiments the assistance of Lyn Magel and O. J. Sibul was especially helpful. Finally our thanks to Mrs. Doris Victory for typing the final paper.

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Model Number	Outside Radius (Ft.)	Bottom Attachment	Draft Ft.	Slenderness Ratio	Vertical Prismatic Coefficient	C. G. From Bottom (Ft.)	C. B. From Bottom (Ft.)	Radius of Gyration (Ft.)	Weight	Cone Height (Ft.)	Techniques Applied & (Wave Cond.)	Pressure Measurement & Flow Visualization
1	.0833'	Cone									Multiple Flash Photograph Tech. (Regular Wave)	
		Flat Disk										
2	.14583'	Cone	2.52	17.3	.93	1.02	1.34	.8738	9.80		Motion Transducer Motion Picture Tech. (Regular Waves)	
		Flat Disk										
3	.1875'	Cone	1.8796'	10		.790	1.0221	.642	11.78	.25	Accelerometer & Rate Gyro Tech. (Regular Waves - Irregular Waves)	(Press. Me.) (Flow Visualization)
		Flat Disk	1.8783'	10	1	.728	.9392	.608	12.90		Accelerometer & Rate Gyro (Regular)	(Flow Visualization)
4	.2318'	Cone								.25	"	
		Flat Disk	1.167'	5.05	1	.4580	.5919	.4135	12.46		"	

TABLE 1.

Free Oscillation Experiment

Model No.	Bottom Attachment	HEAVE				PITCH		
		T_o (Sec.)	$(KH)_o$	$\frac{K \text{ measured}}{K \text{ calculated}}$	$\overline{K} = \frac{K \text{ measured}}{\omega_o}$	T_o (Sec.)	$(KH)_o$	$\overline{K}_m = \frac{K_m}{\omega_o}$
3	Cone	1.493	1.05	7.43	.00998	1.812	.701	.01915
3	Flat Disk	1.58	.925	15.77	.01785	1.864	.6625	.0394
4	Flat Disk	1.3	.86	5.11	.0237	1.64	.540	.0462

K: damping coefficient

$\overline{K}$: non-dimensional damping coefficient

TABLE 2

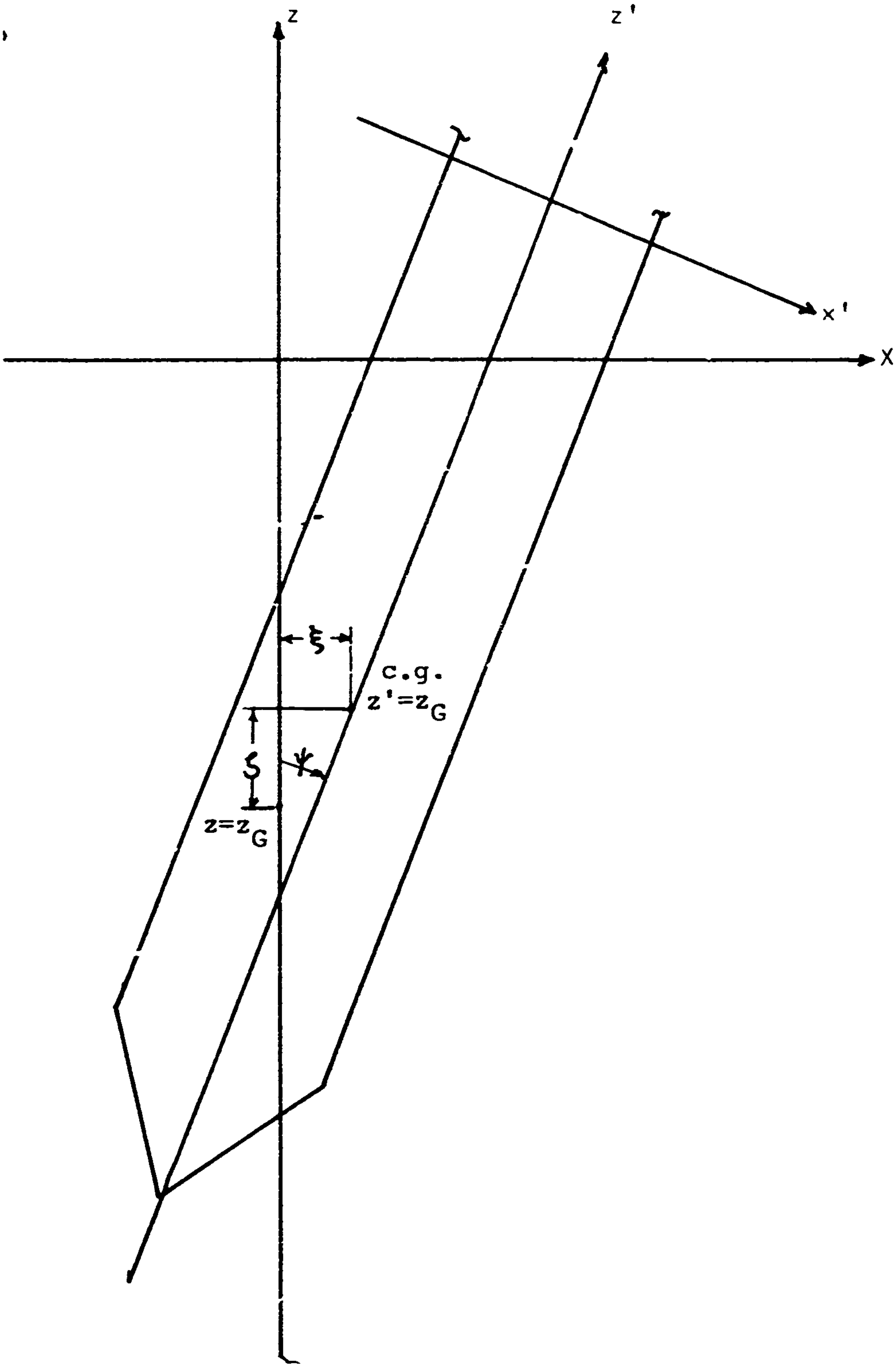


FIGURE 1. Coordinate Systems

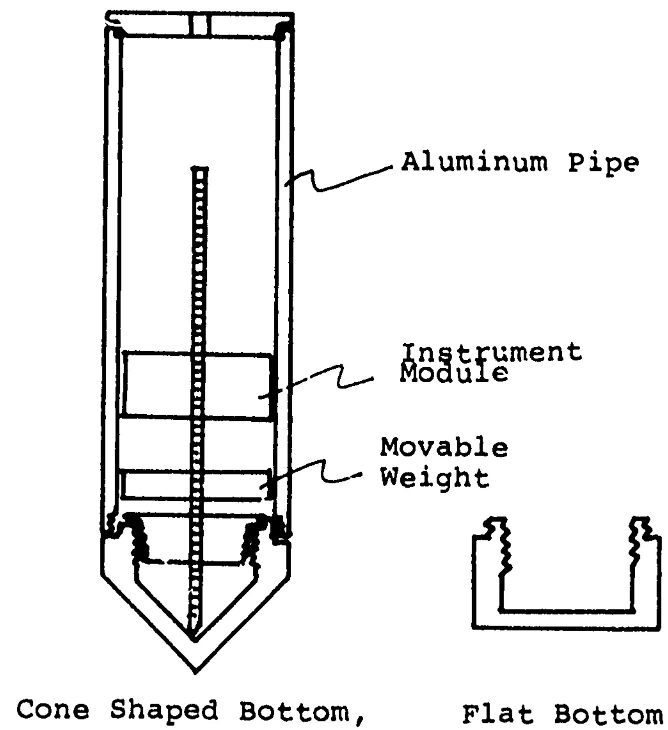


FIGURE 2. Model Construction

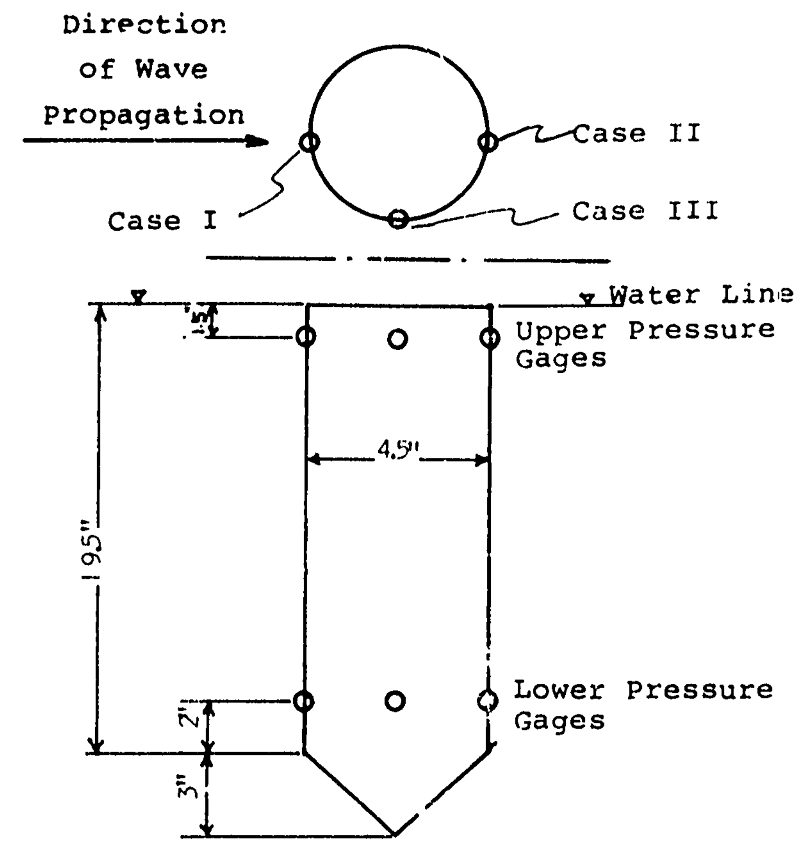


FIGURE 3. Position of Pressure Gages

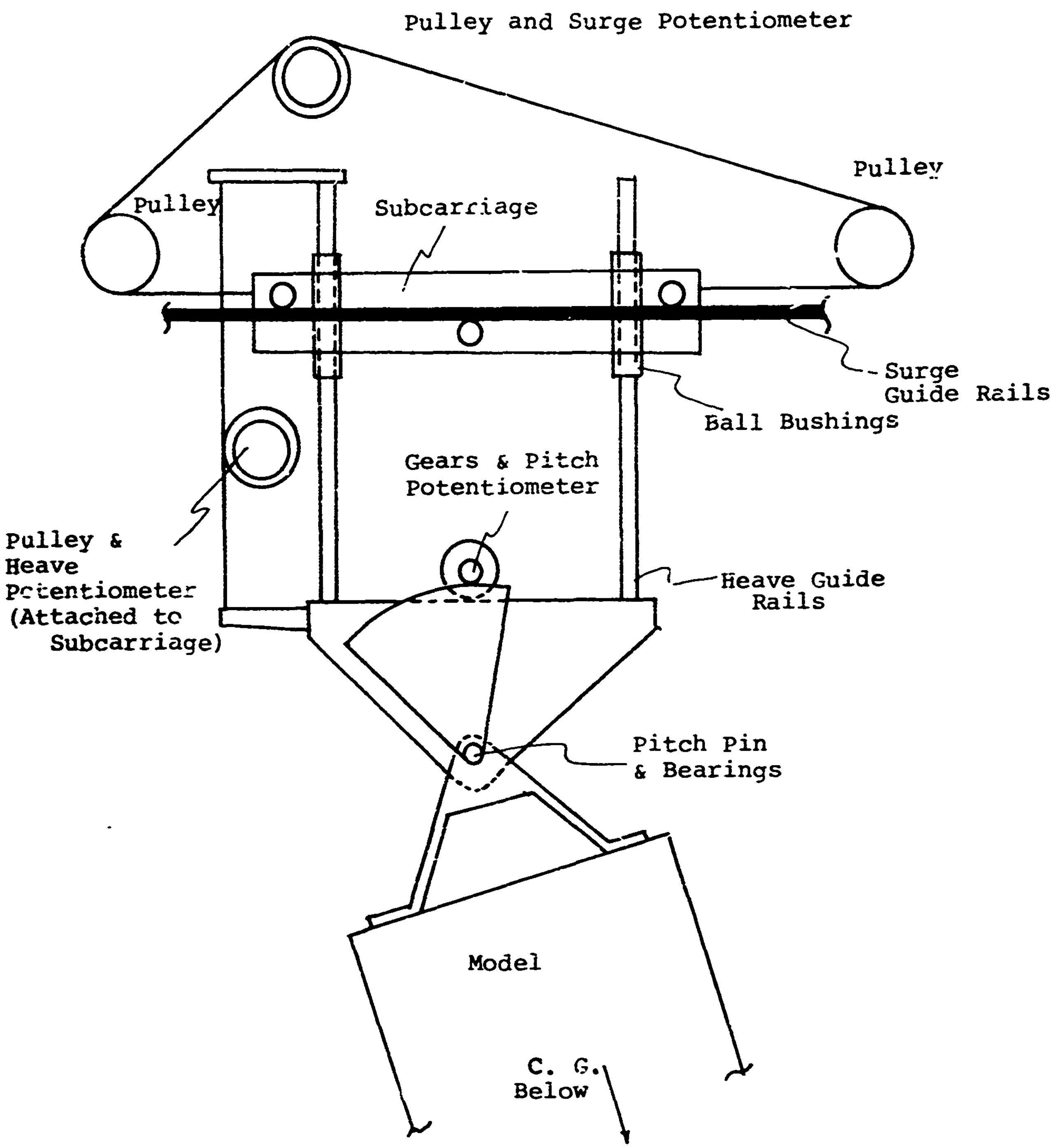
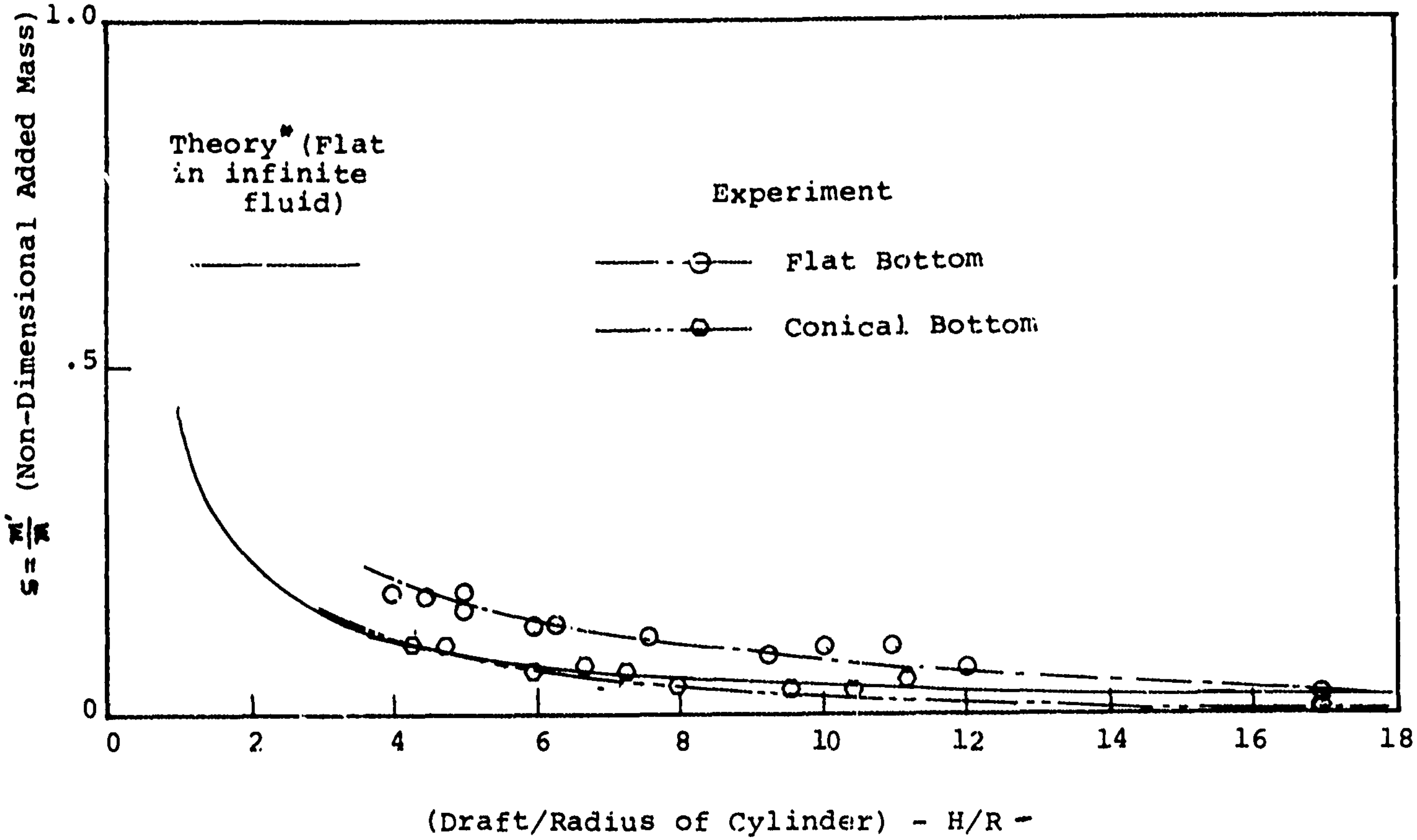


FIGURE 4. Schematic of Motion Transducer Apparatus



*One half of the added mass of a disk in infinite fluid: $S = \frac{m'}{m} = \frac{4}{3\pi} \frac{1}{(H/R)}$
(See Lamb's "Hydrodynamics", p. 144)

FIGURE 5. Non-Dimensional Added Mass vs. Slenderness Ratio

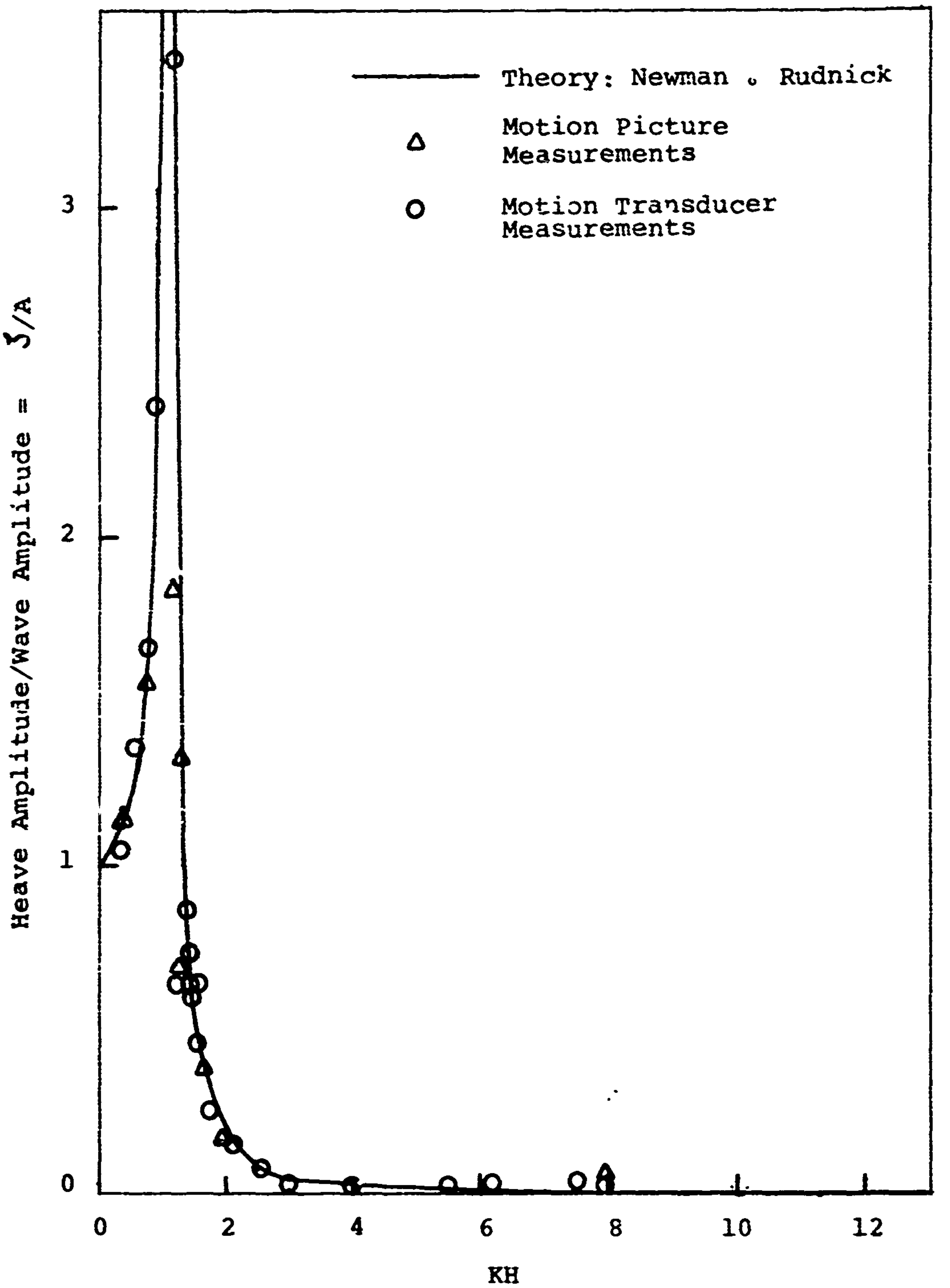


FIGURE 6. Heave Response for Model #2 with Conical Bottom

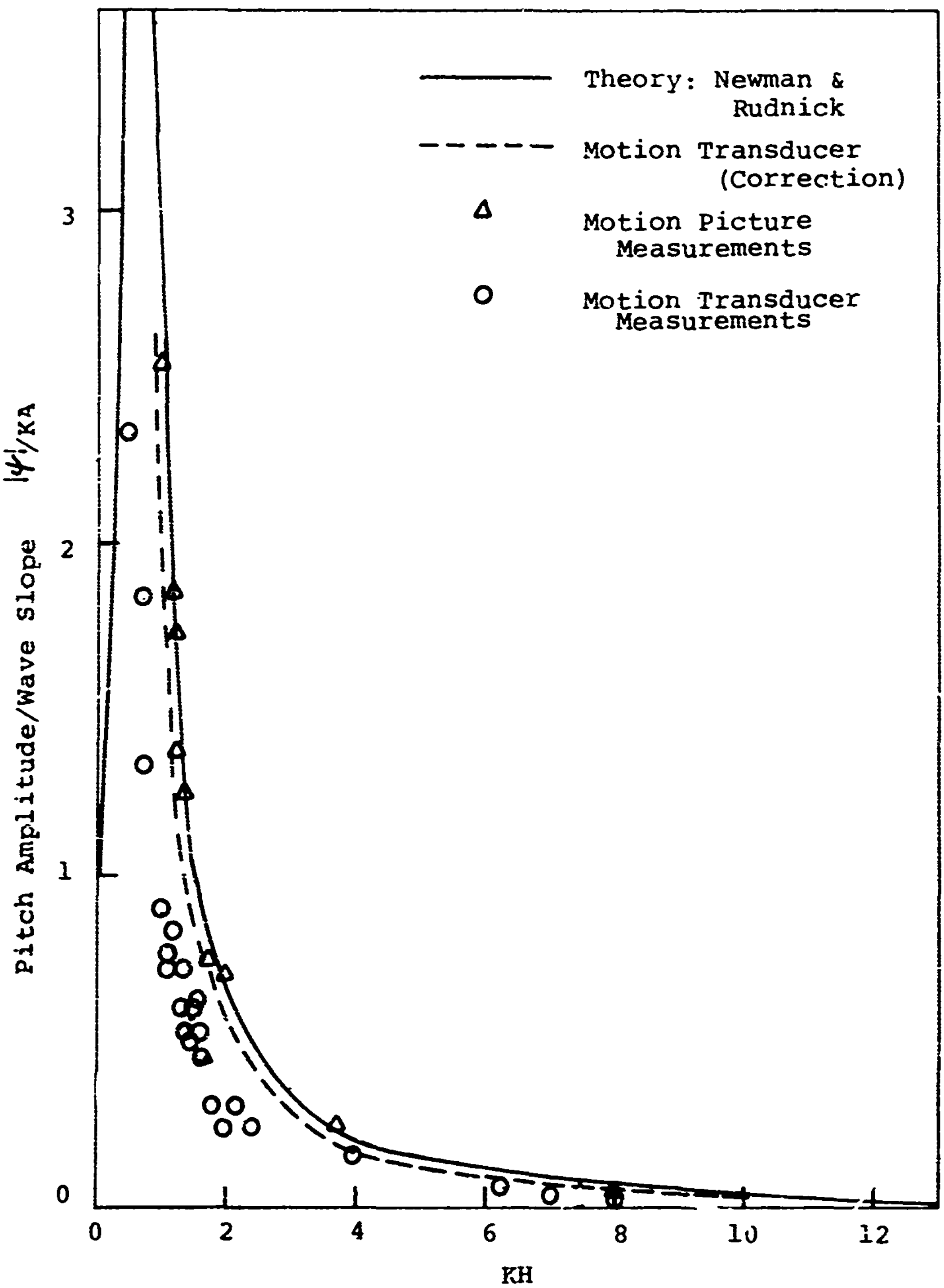


FIGURE 7. Pitch Response for Model #2 with Conical Bottom

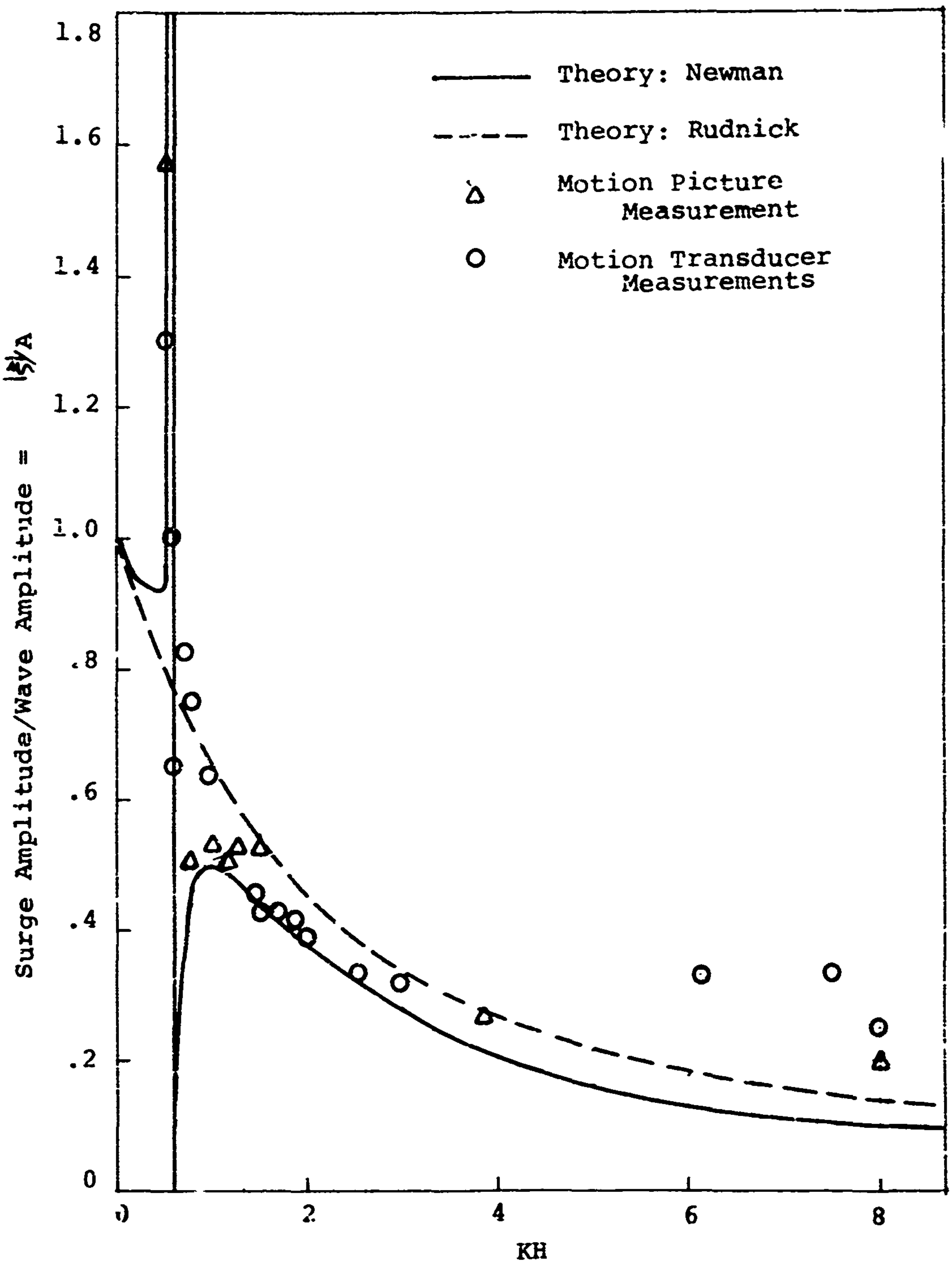


FIGURE 8. Surge Response for Model #2 with Conical Bottom

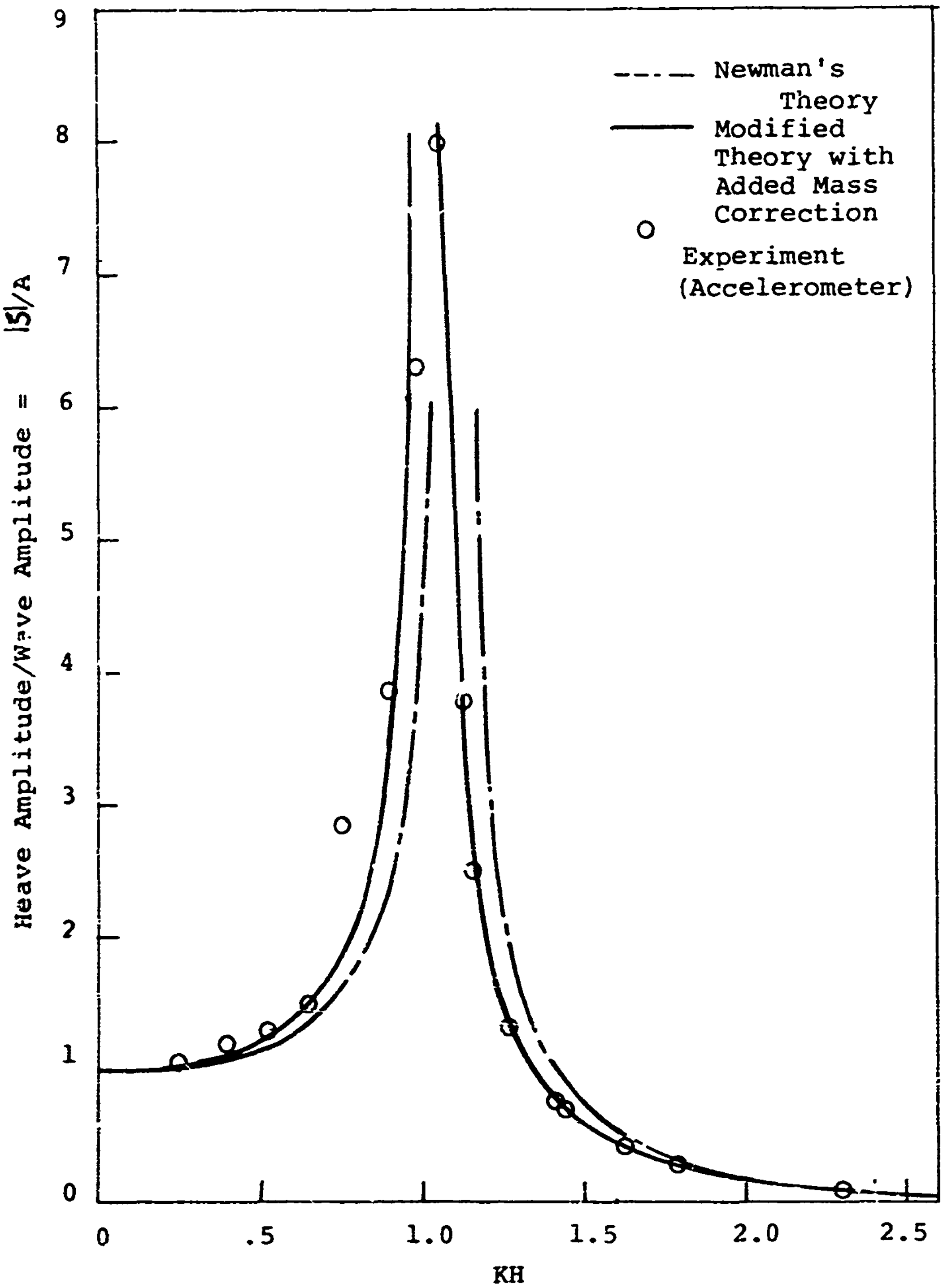


FIGURE 9. Heave Response for Model #3 with Conical Bottom

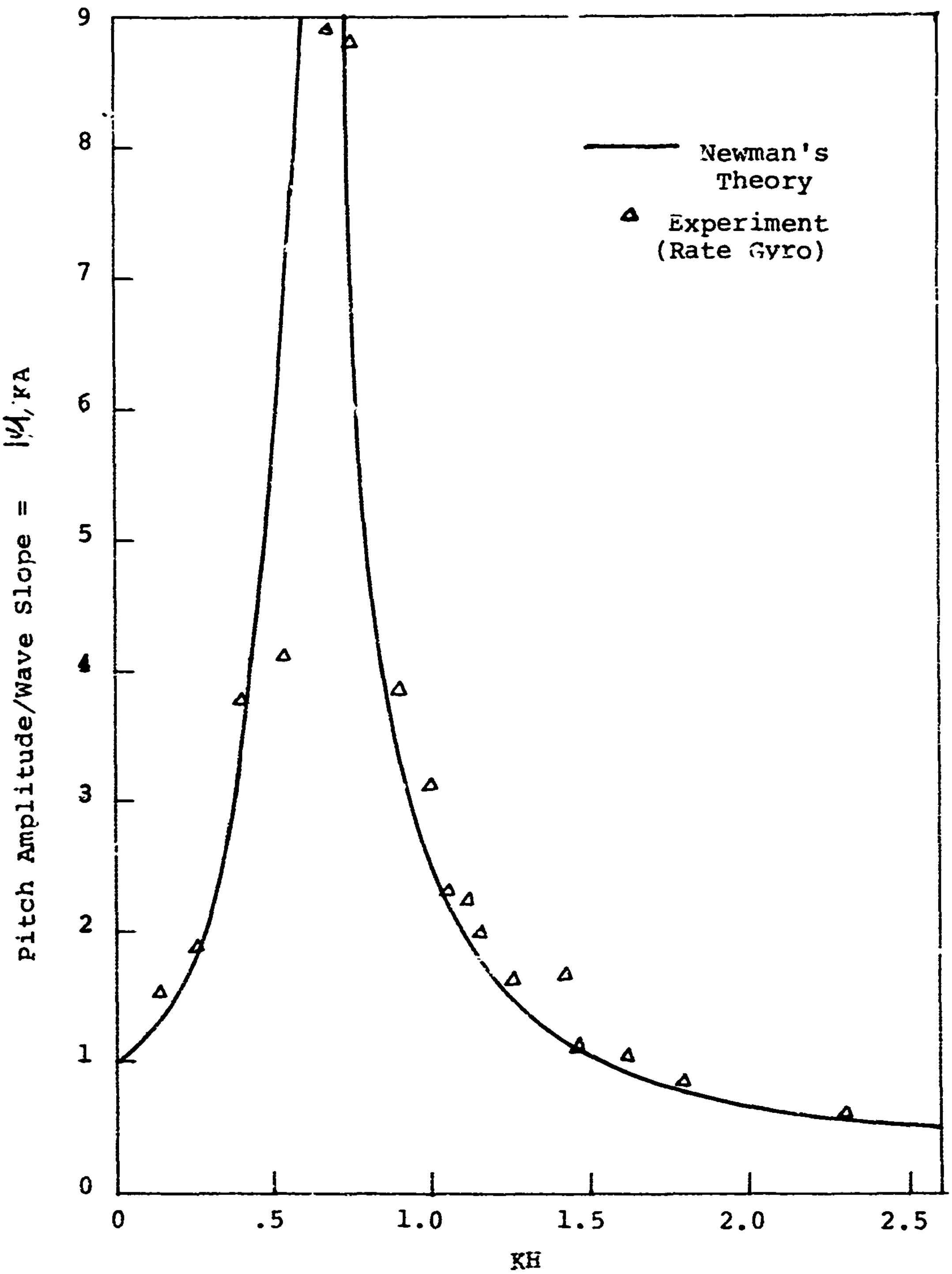


FIGURE 10. Pitch Response for Model #3 with Conical Bottom

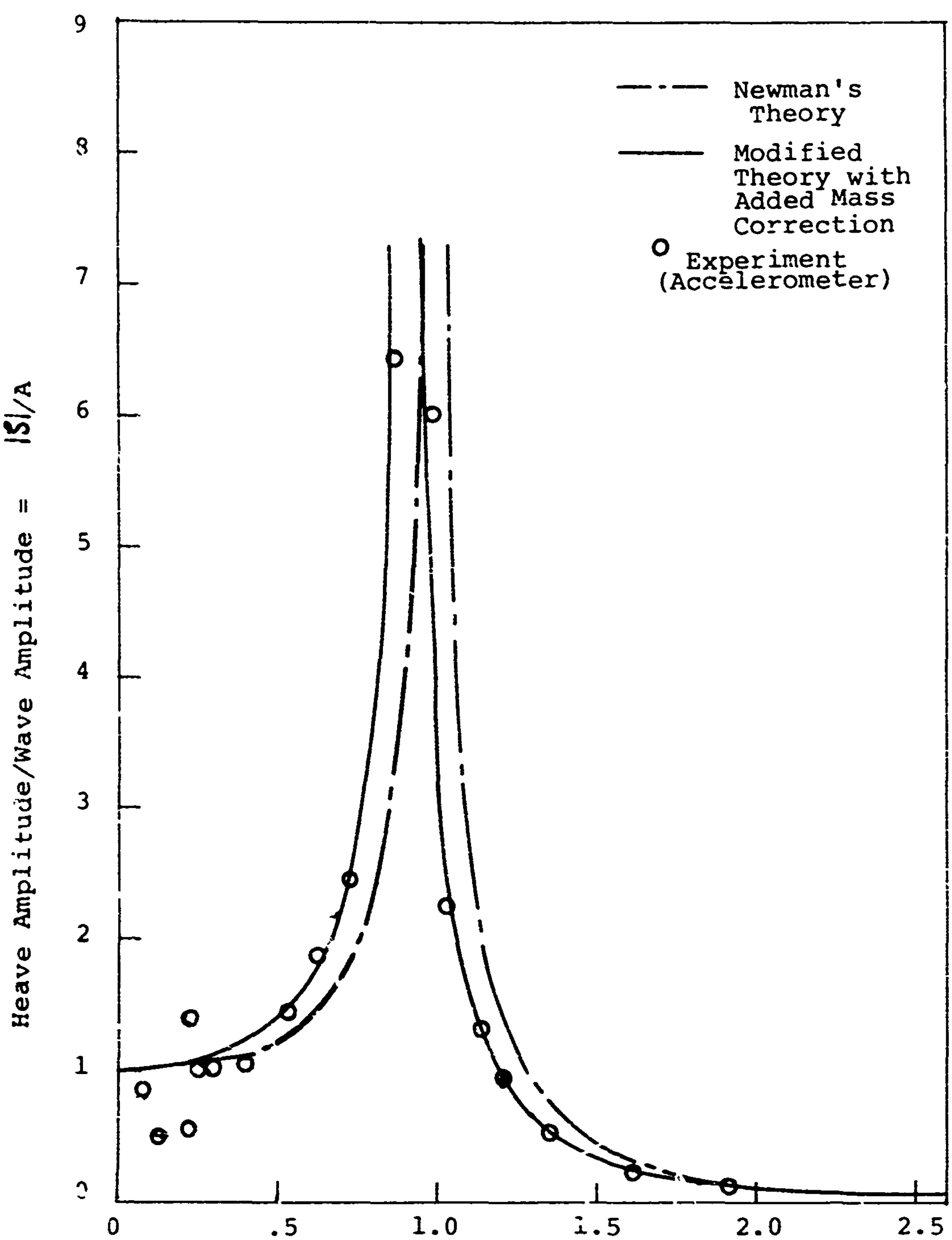


FIGURE 11. Heave Response for Model #3 with r at Bottom

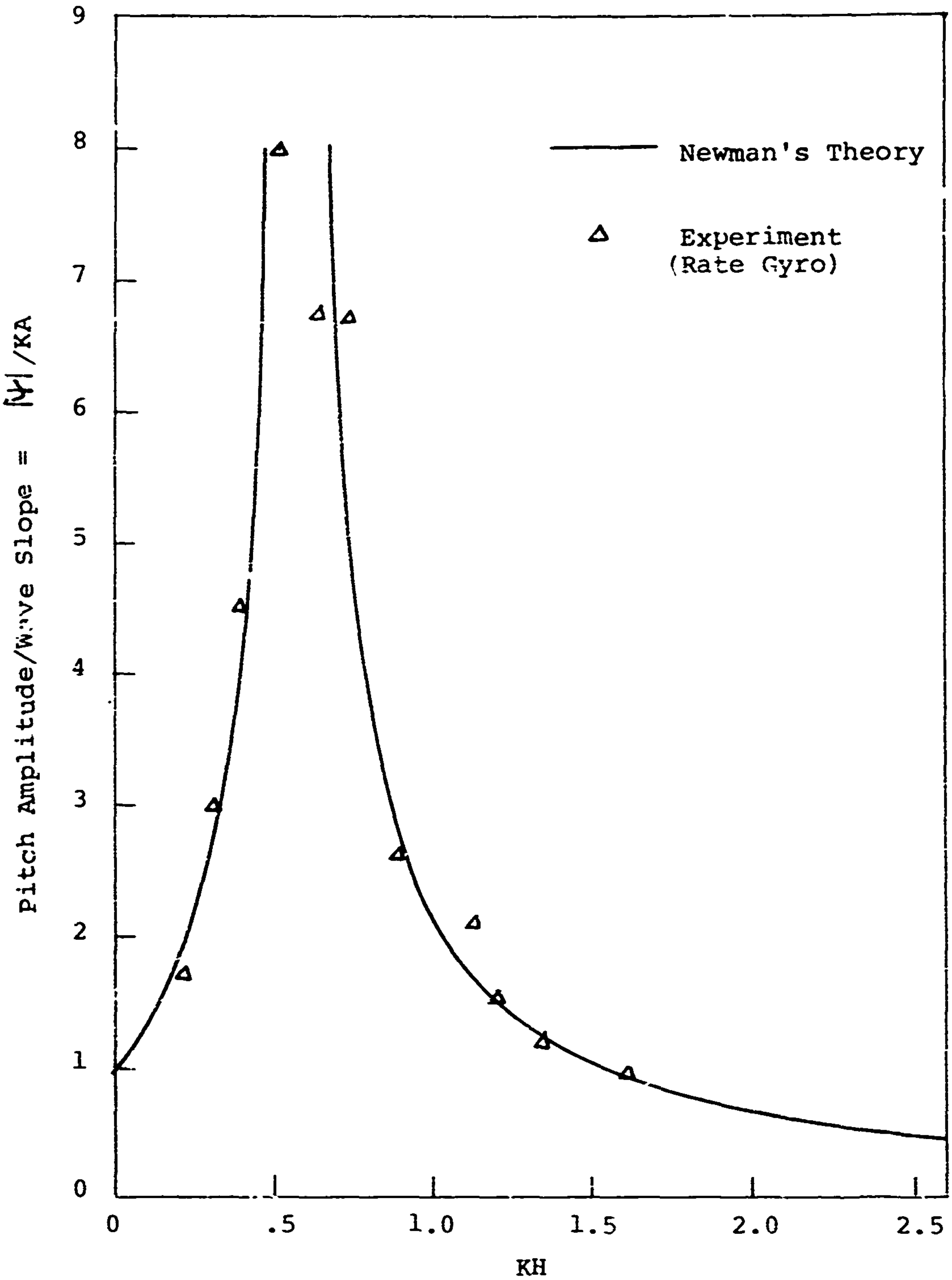


FIGURE 12. Pitch Response for Model #3 with Flat Bottom

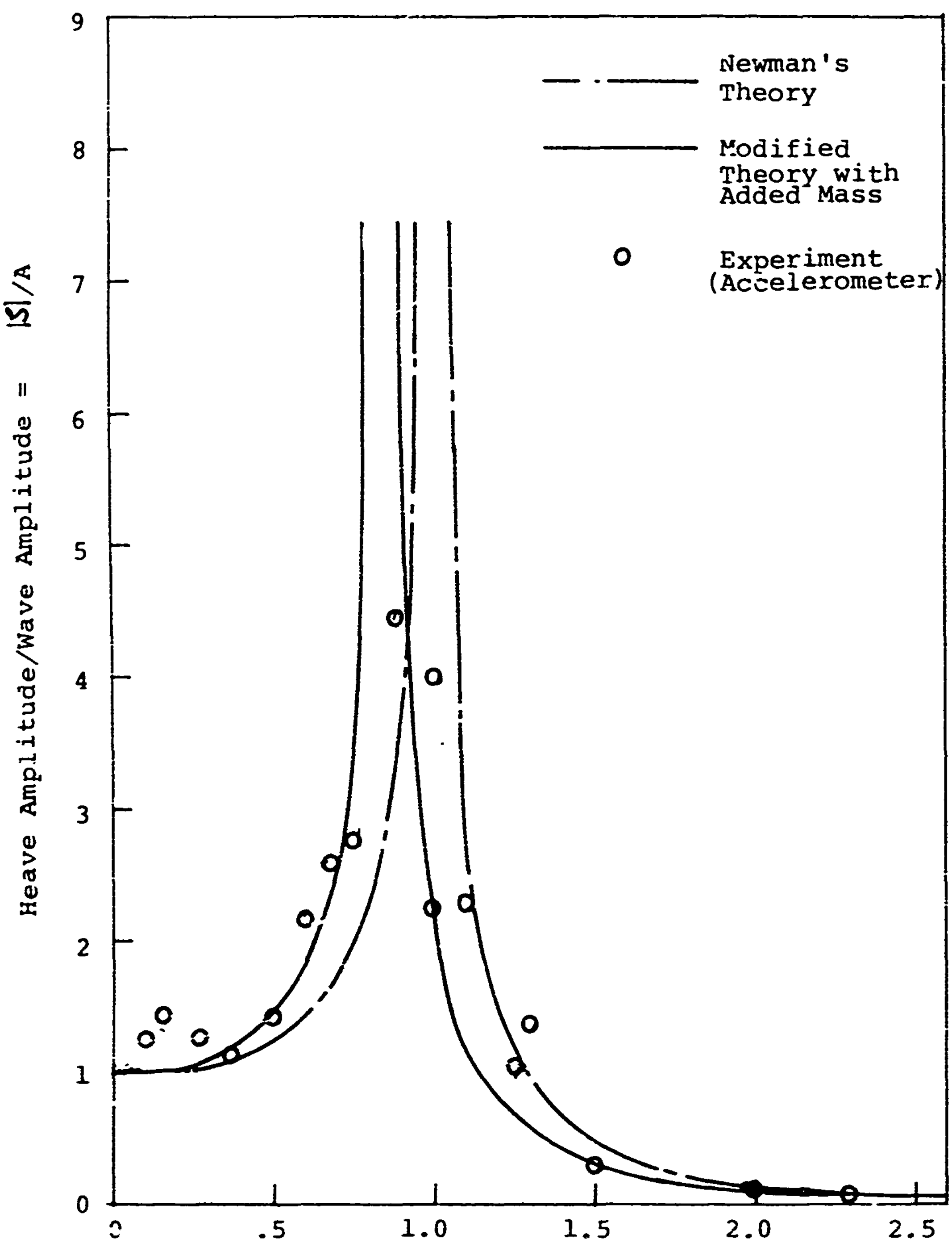


FIGURE 13. Heave Response for Model #4 with Flat Bottom

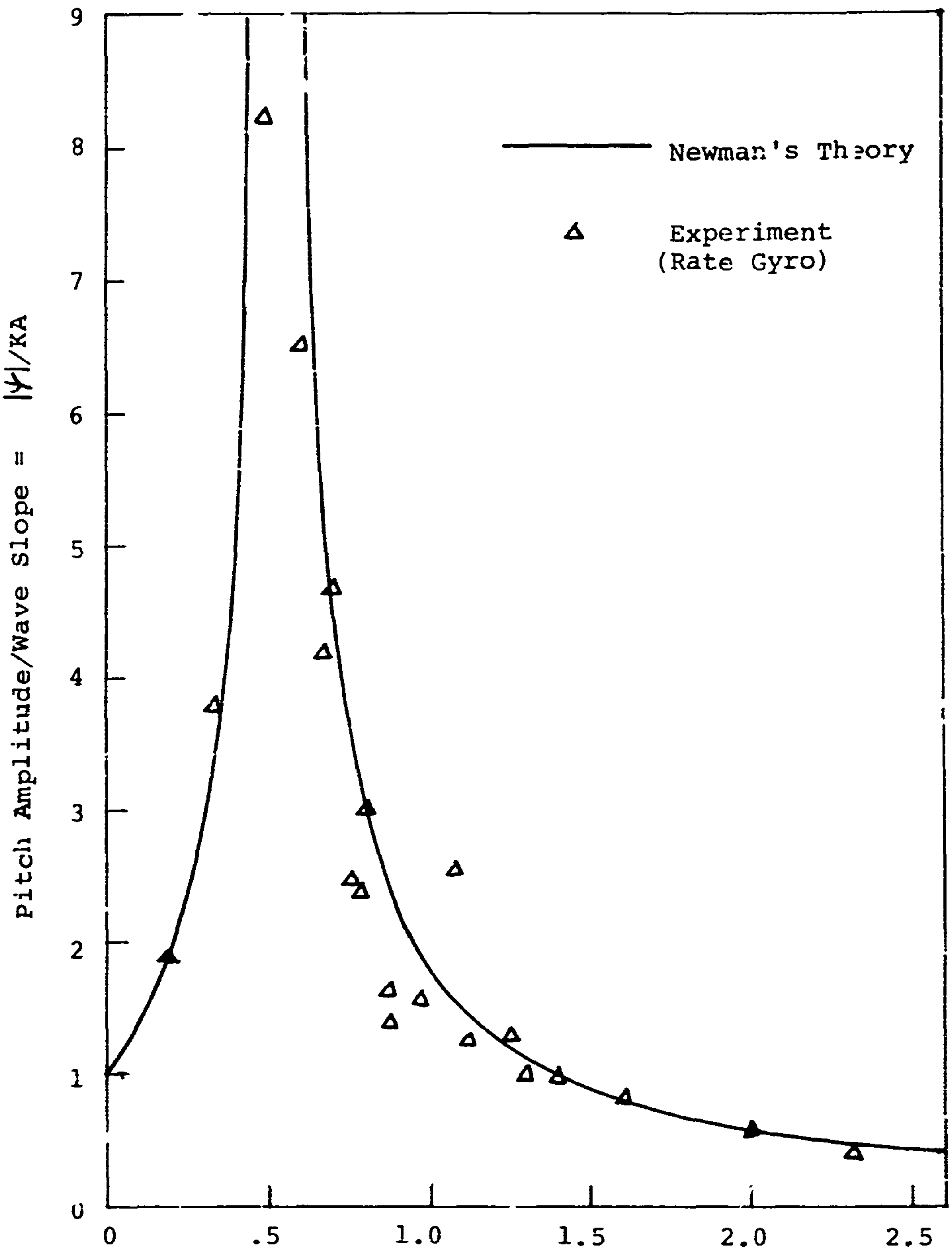


FIGURE 14. Pitch Response for Model #4
with Flat Bottom

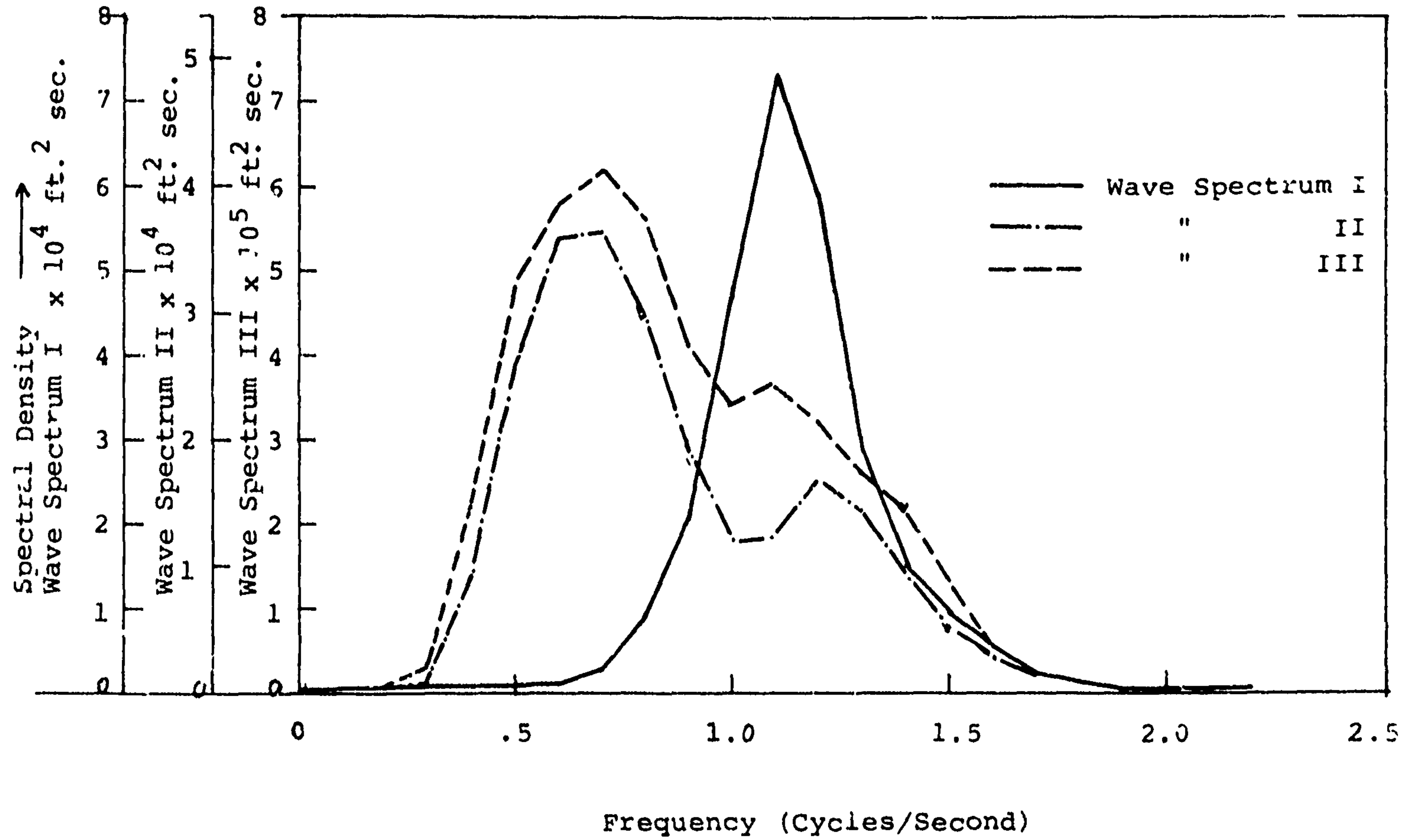


Figure 15. Wave Spectra

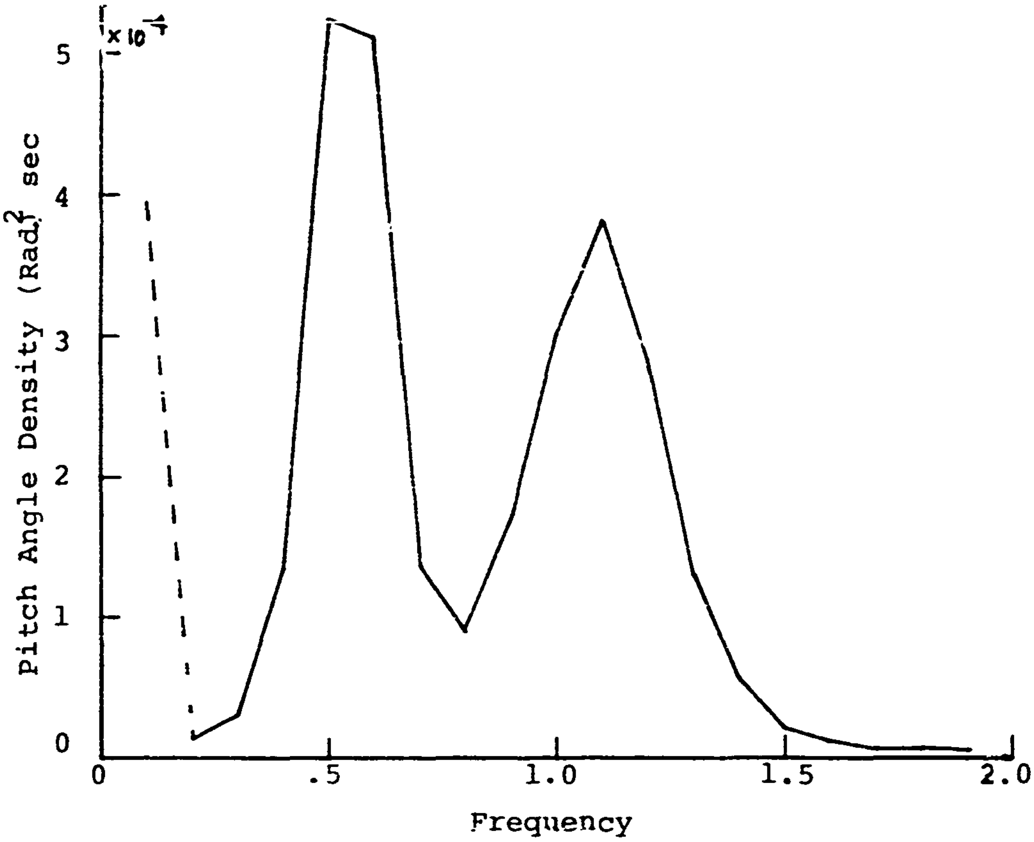


FIGURE 16. Pitch Spectrum for Wave #1
(Angular Displacement)

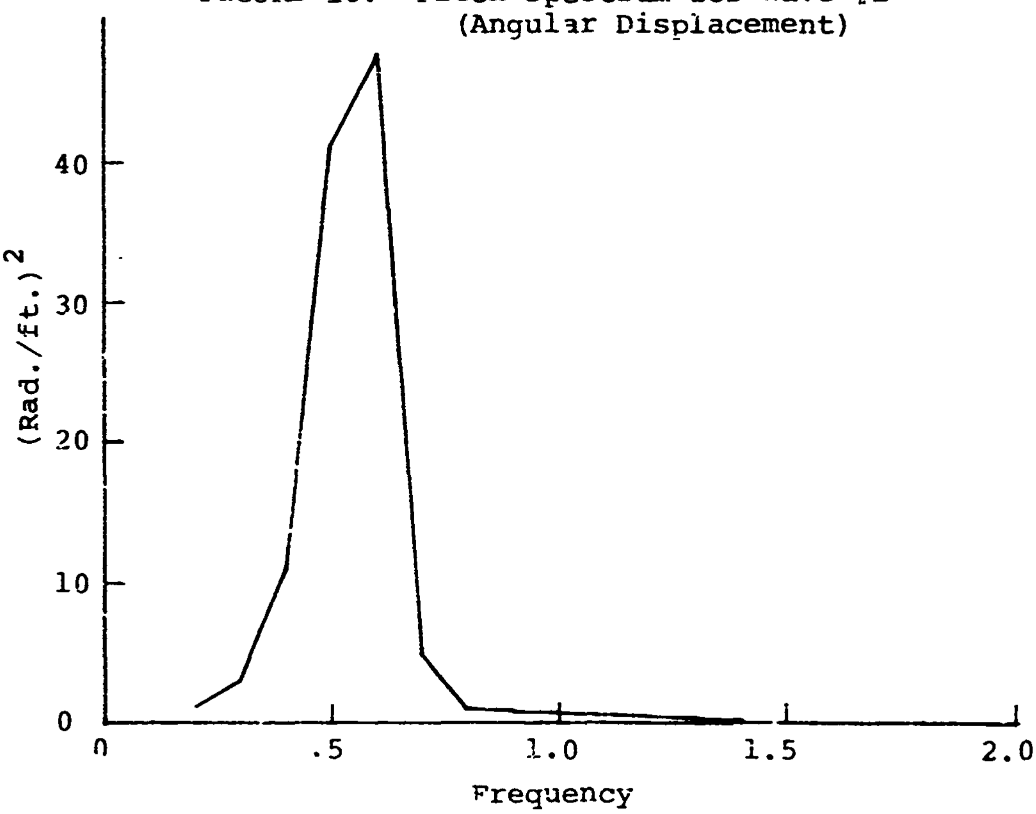


FIGURE 17. Pitch Transfer Function for Wave #1

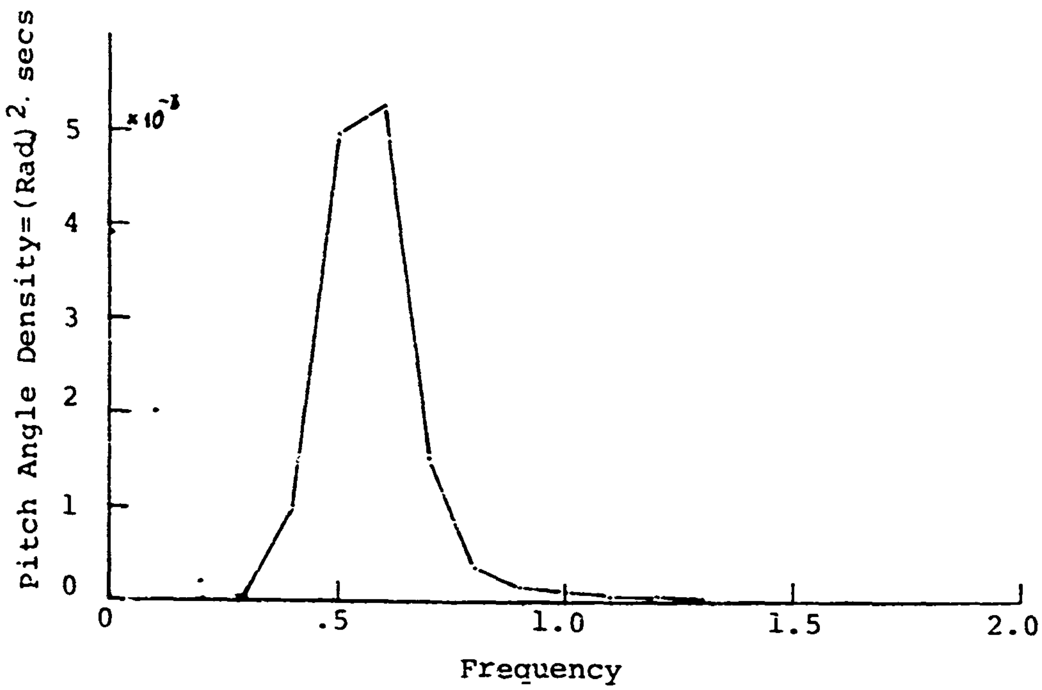


FIGURE 18. Pitch Spectrum for Wave #2
(Angular Displacement)

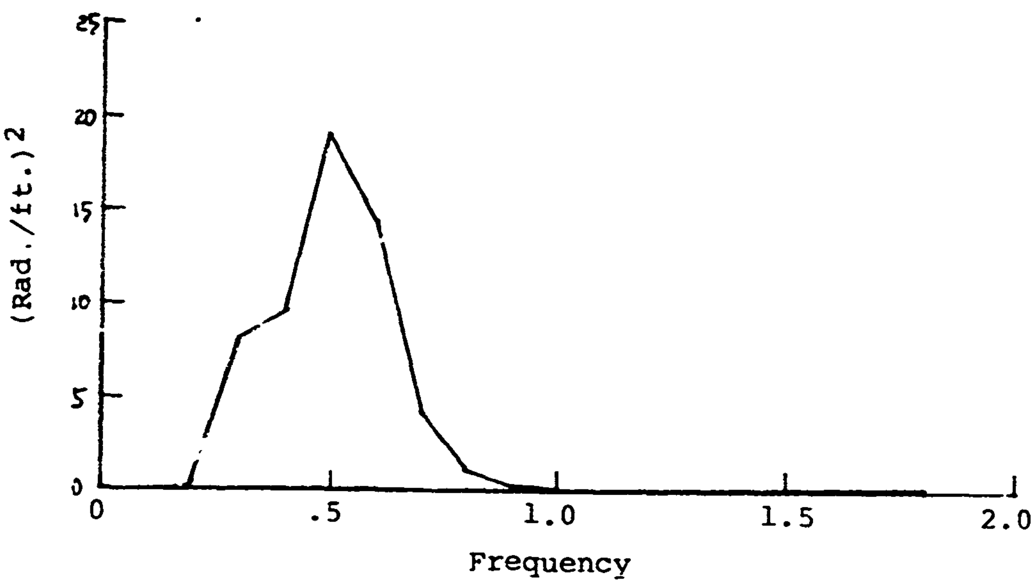


FIGURE 19. Pitch Transfer Function for Wave #2

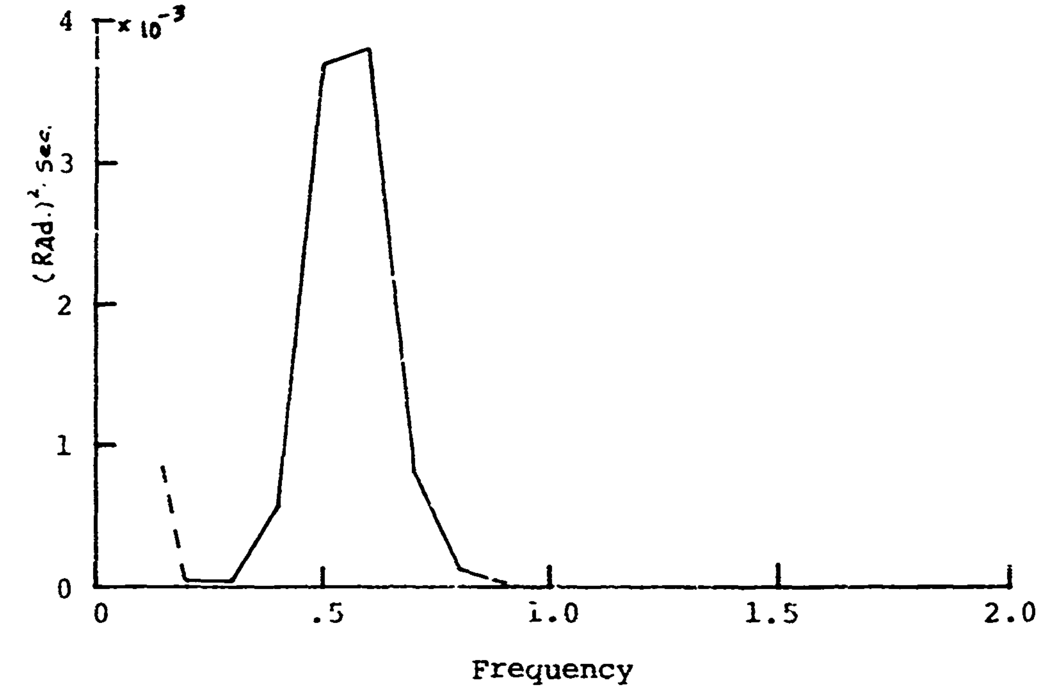


FIGURE 20. Pitch Spectrum for Wave #3
(Angular Displacement)

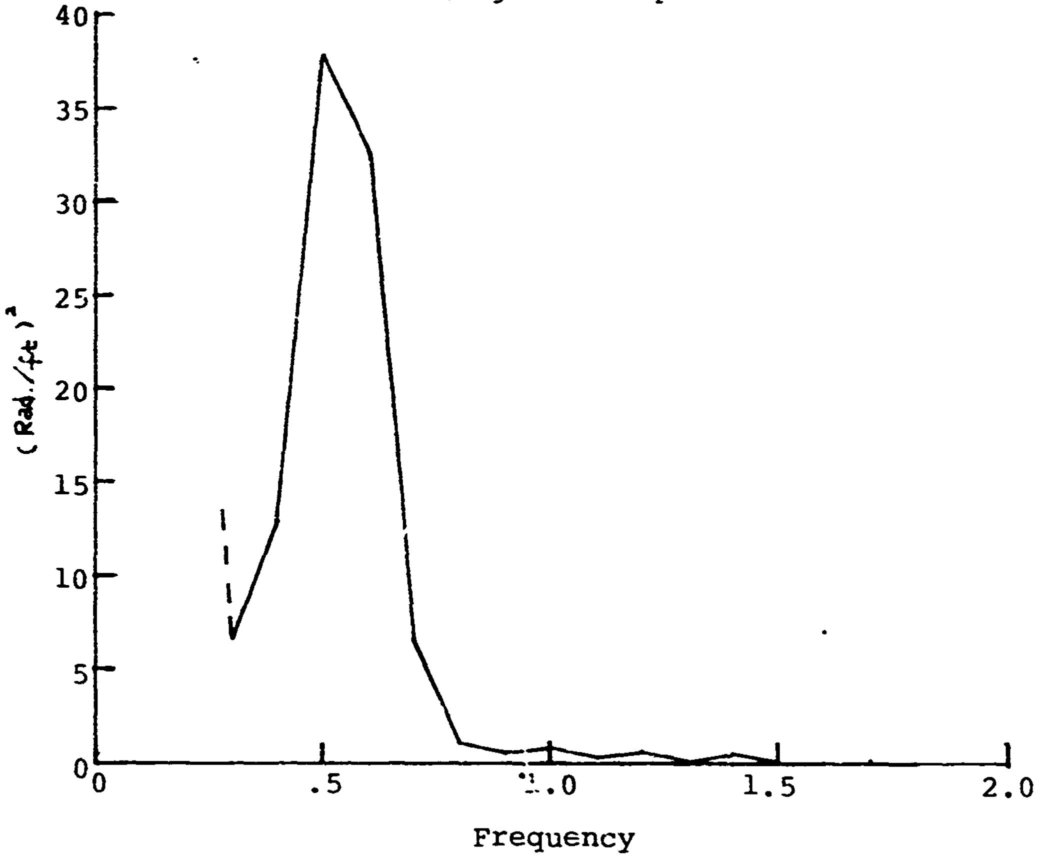


FIGURE 21. Pitch Transfer Function for Wave #3

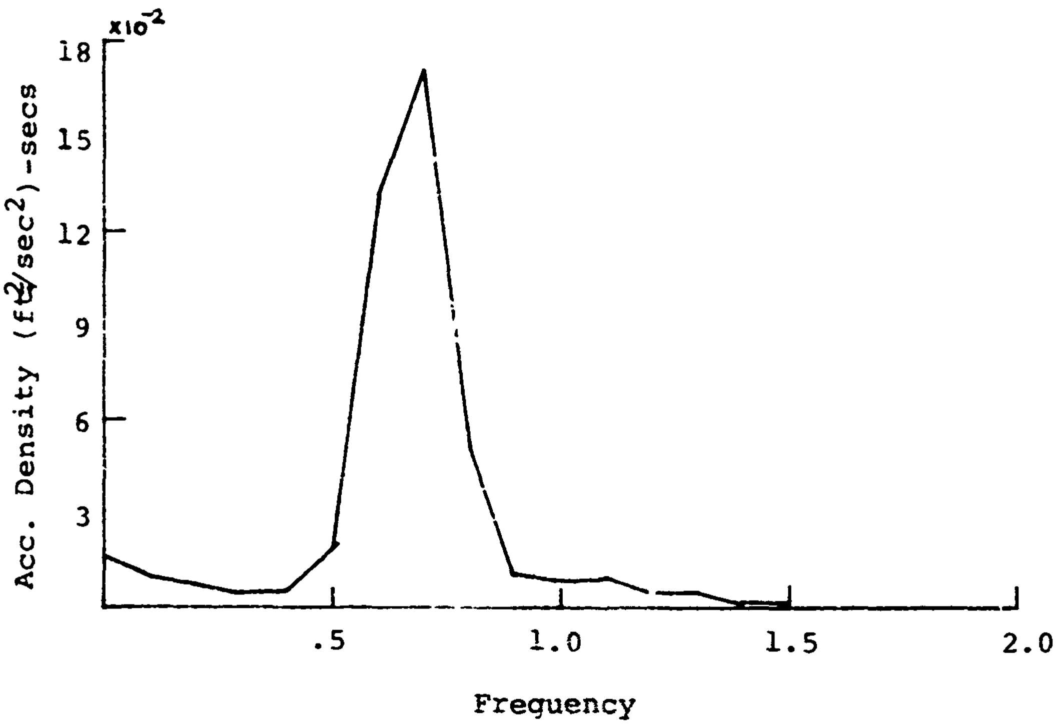


FIGURE 22. Heave Acceleration Spectrum for Wave #1

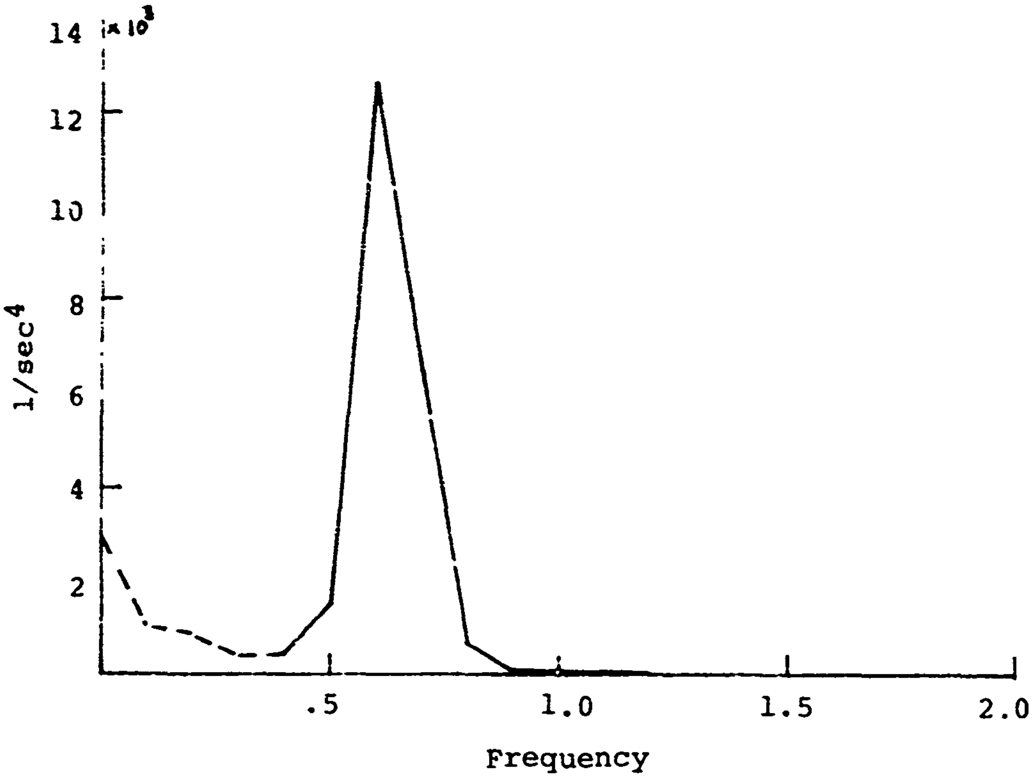


FIGURE 23. Heave Acceleration Transfer Function for Wave #1

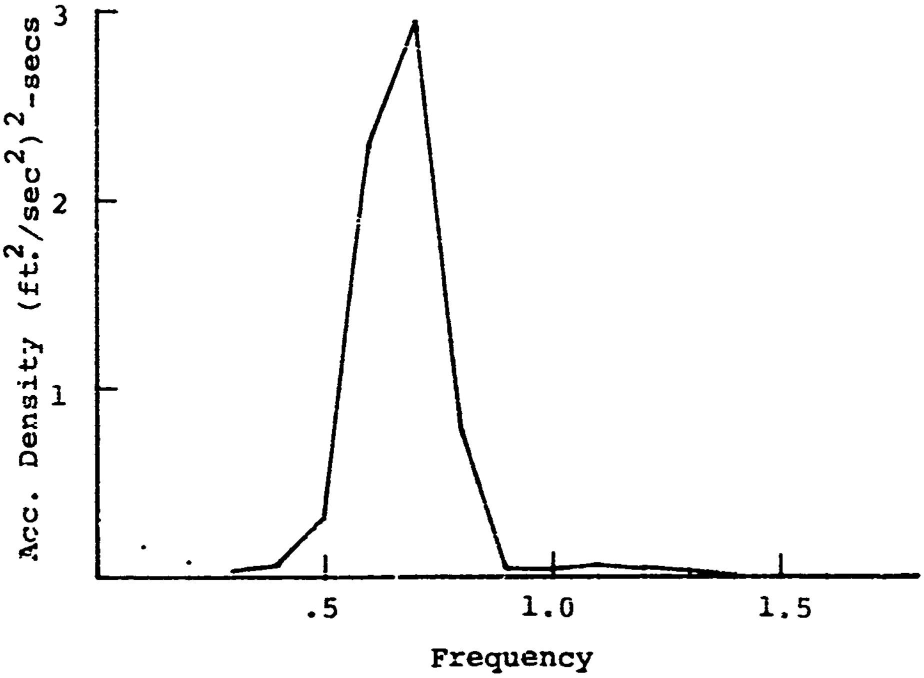


FIGURE 24. Heave Acceleration Spectrum for Wave #2

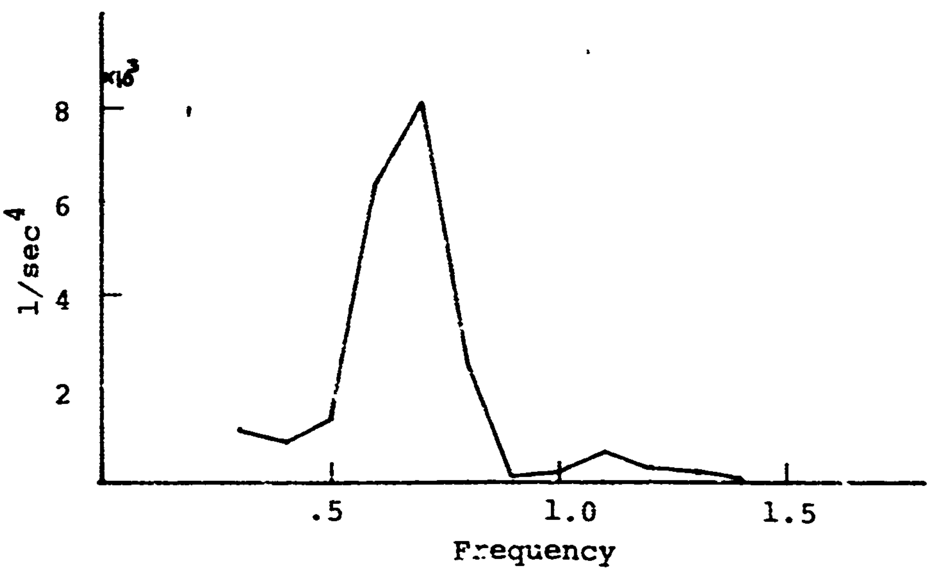


FIGURE 25. Heave Acceleration Transfer Function for Wave #2

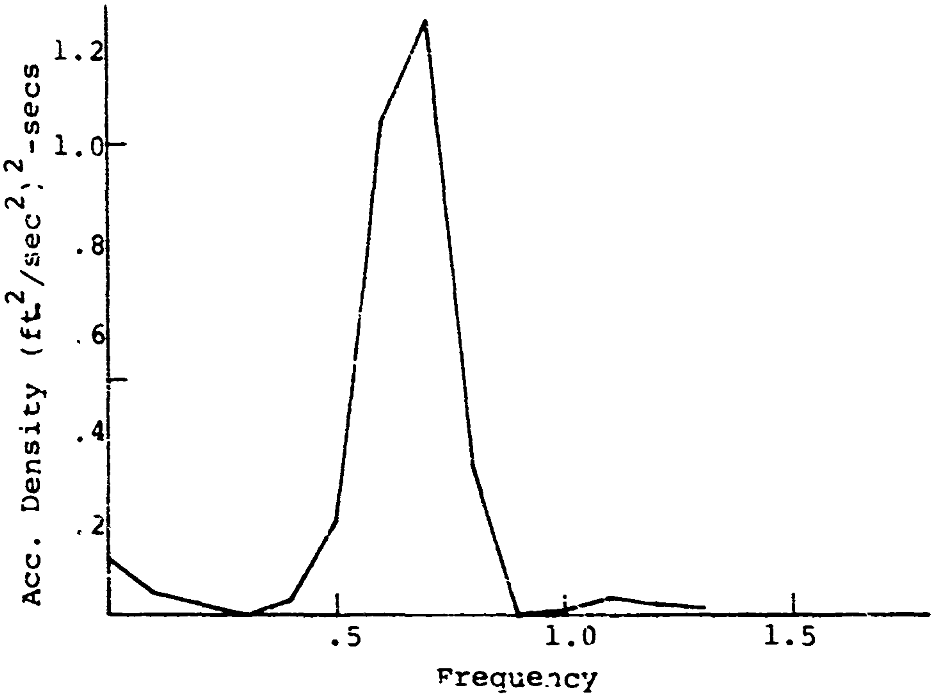


FIGURE 26. Heave Acceleration Spectrum for Wave #3

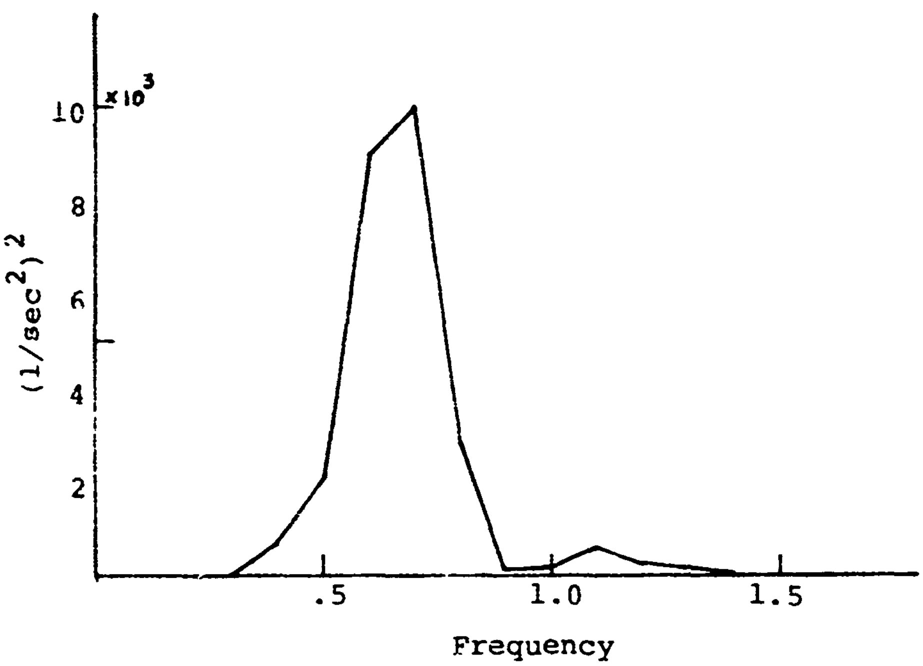


FIGURE 27. Heave Acceleration Transfer Function for Wave #3

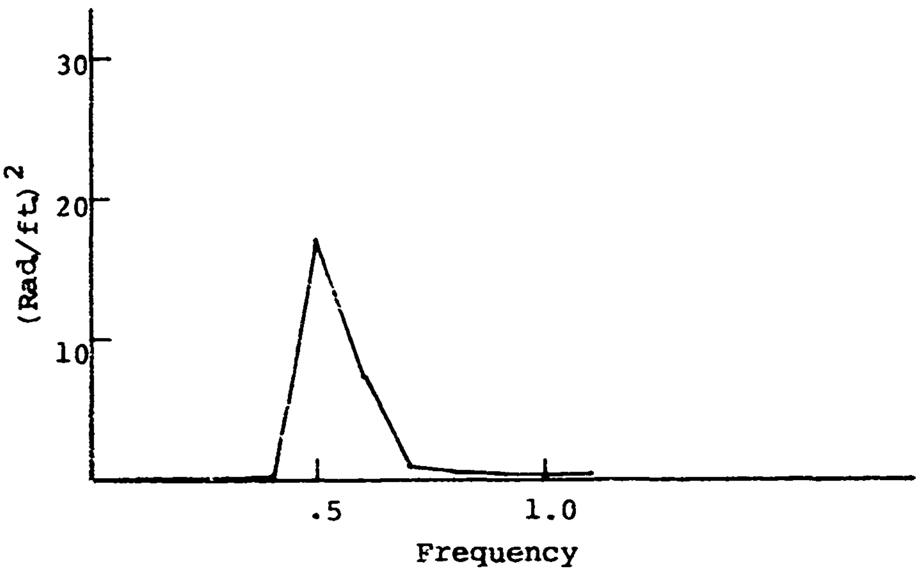


FIGURE 28. Pitch Transfer Functions by Newman's Theory

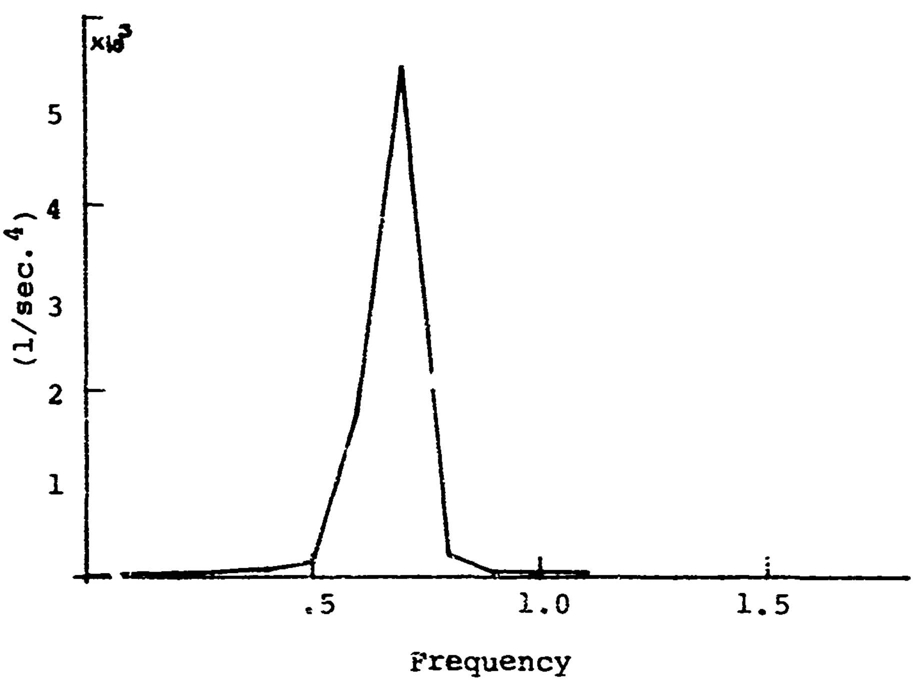


FIGURE 29. Heave Acceleration Transfer Functions by Newman's Theory

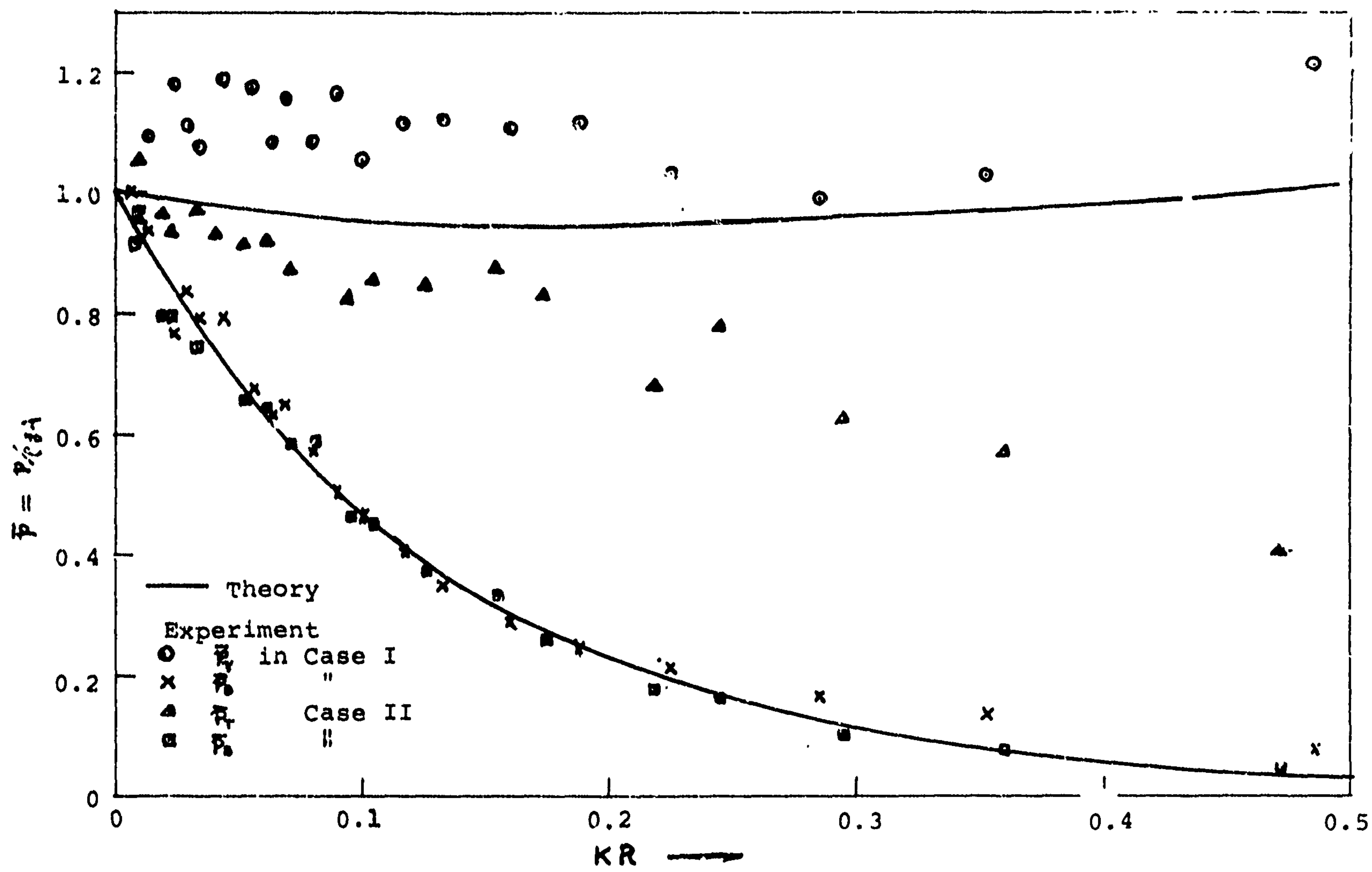


FIGURE 30. $\bar{p}$ vs. KR for Case I & II

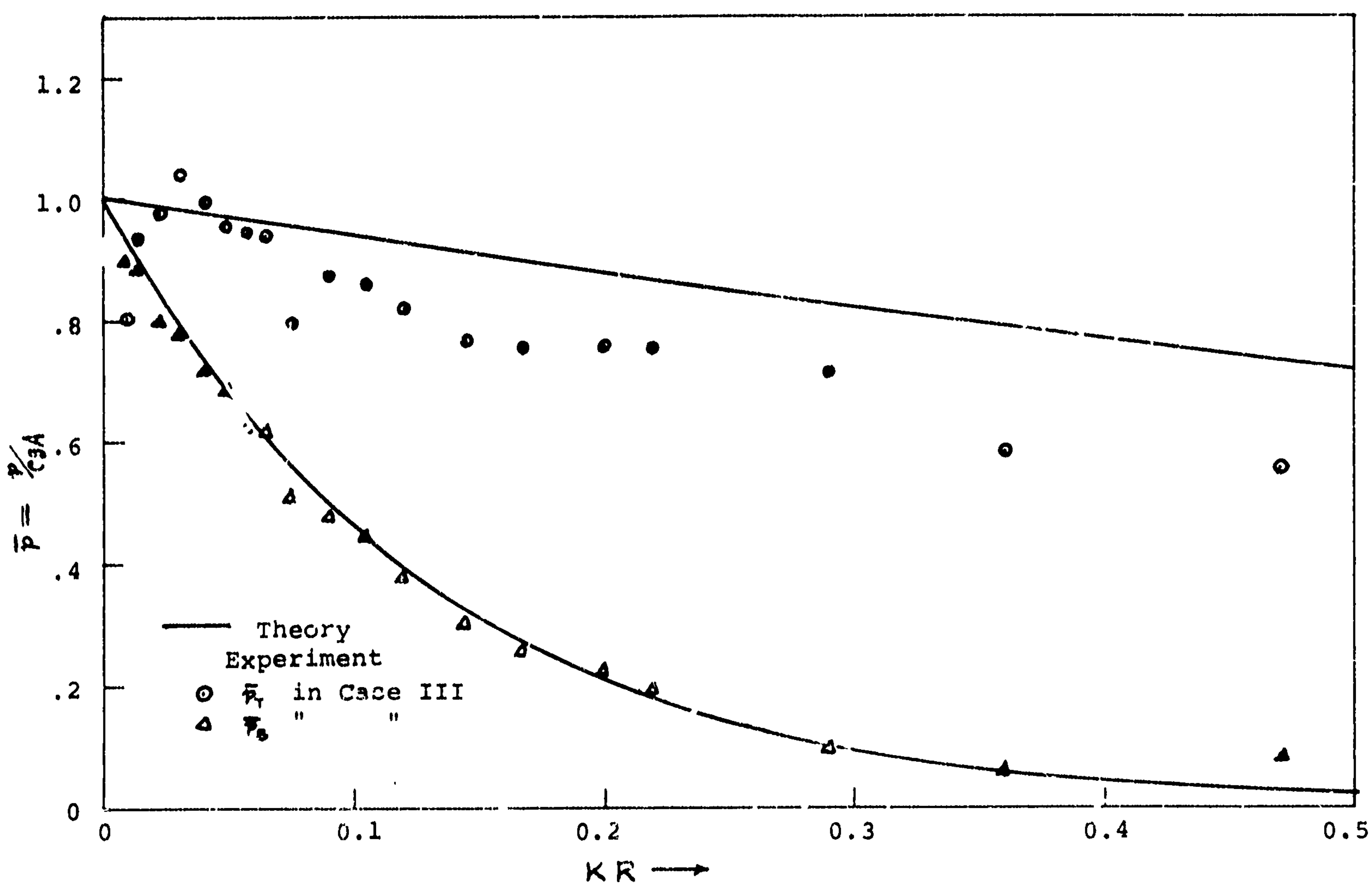
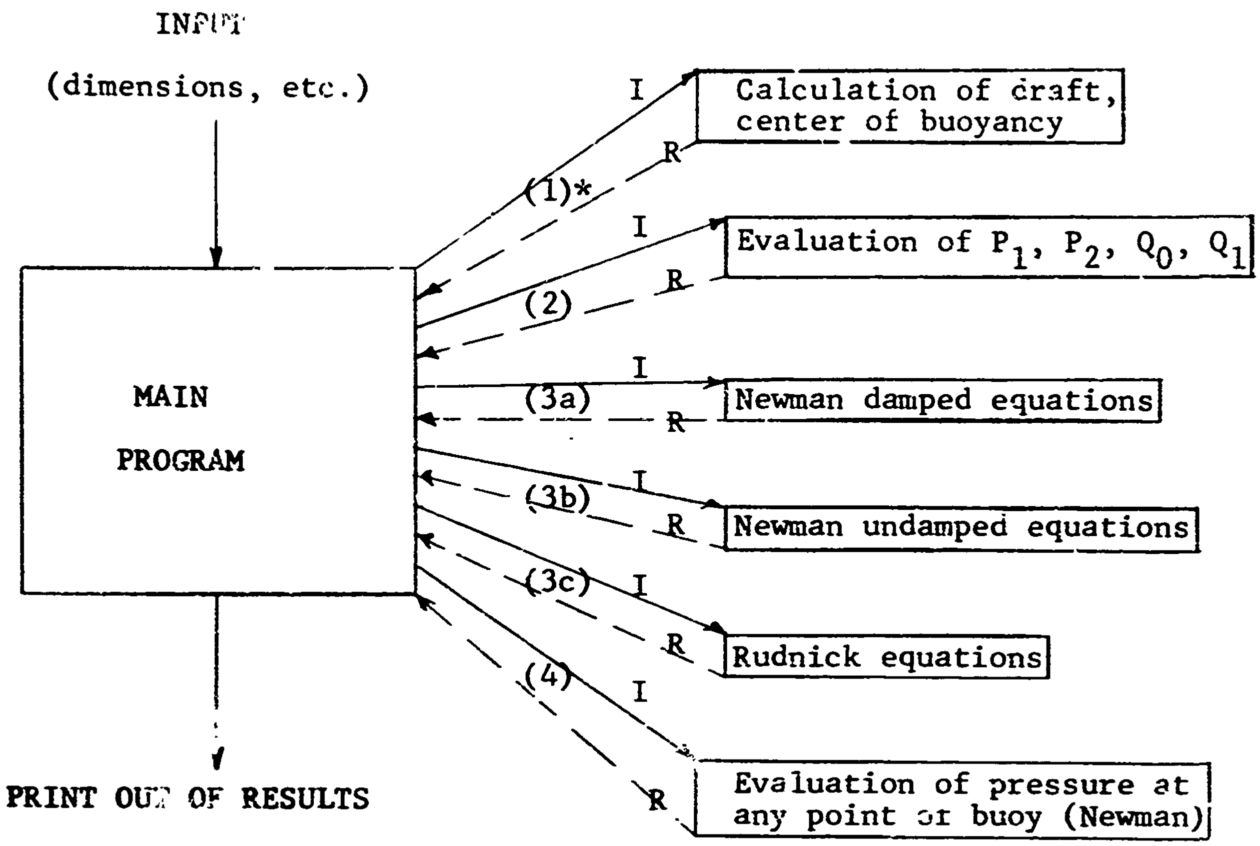


FIGURE 31. $\bar{p}$ vs. KR for Case III



I = information needed for computations of subprogram

R = results of subprogram computations

<u>Possible shapes for which calculations can be made</u>:

(1) Cylindrical segments
(2) Conical Segments
(3) Hemispherical ends
(4) Elliptical ends
(5) Third degree polynomial end

* The numbers in parentheses in the diagram indicate the steps involved.

FIGURE 32. Schematic Flow Diagram of Computer Program

APPENDIX A: SPAR COMPUTER PROGRAM AND ITS WRITE-UP

```
      PROGRAM SPAR (INPUT,OUTPUT)
      DIMENSION NOSECT(8), NTYPE(8), AZ(9),AAA(8),BBB(8),CCC(8),DDD(8)
      DIMENSION ZETA(50), UNZETA(50), XI(50), UNXI(50), PSI(50)
      DIMENSION ZETA1(50),ZETA2(50),ZETA3(50),ZETA4(50)
      DIMENSION UNPSI(50)
      DIMENSION Q0(50), Q1(50), Q00(50), Q11(50)
      DIMENSION AK(50)
      DIMENSION DAMP(50)
      DIMENSION DAMP1(50)
      DIMENSION HH(8)
      DIMENSION AKH(50)
      DIMENSION PSI3(50), XI3(50)
      DIMENSION TITLE(10)
      DIMENSION EPSILO(50)
      DIMENSION AMOM(8)
      DIMENSION CFREQ(50), RPSI(50)
      COMMON/FIRST/A, B, C, D, I, L, SUBVOL, AREA, TOTVOL, VOL, H, CHI,
     1     W, G, ROE, SG, AZ, AMOM, HH, ADD
      COMMON/SECOND/BB, Z1, Z0, P11, P1, ZG, P22, P2, N, K, AW, AB, AA,
     1     CA, Z, AK, Q00, Q0, Q11, Q1
       COMMON/THIRD/QQ, QR, OMEGA, AR, ZETA, PSI, XI, EPSILO,
     1     UNZETA, UNPSI, UNXI, ZETA1, ZETA2, ZETA3, ZETA4, XI3, PSI3
      COMMON/FOURTH/NTYPE, AAA, BBB, CCC, DDD
      COMMON/FIFTH/TITLE, HEVKH, HEVK, HEF, BG, SQAR,
     1     MOVER, CFREQ, RPSI, AKH
C FIRST DATA CARD IS TITLE CARD, ALL 80 SPACES ARE READ
      READ 9, TITLE
    9 FORMAT (10A8)
      READ 10, NOSEC, G, ROE, W, ZGB
   10 FORMAT (I10, 4F10.4)
C NOSEC = NUMBER OF SECTIONS
C G = ACCELERATION OF GRAVITY (FT/SEC**2)
C ROE = DENSITY OF FLUID  (LB-SEC**2/FT**4)
C W = WEIGHT OF MODEL (LB)
C ZGB = POSITION OF CENTER OF GRAVITY MEASURED FROM BASE  (FT)
      VOL = W/(ROE*G)
C VOL = DISPLACED VOLUME REQUIRED BY ARCHIMEDES  (FT**3)
C LIMIT PROGRAM TO 8 SECTIONS
      DO 25 I = 1, NOSEC
      READ 24, NOSECT(I), NTYPE(I), AZ(I+1),AAA(I),BBB(I),CCC(I),DDD(I)
   24 FORMAT (2I10, 5F10.4)
      AMOM(I) = 0.0
   25 CONTINUE
C NOSECT(I) = NUMBER OF EACH SECTION BEGINNING AT THE BASE
C NTYPE = A NUMBER CLASSIFYING THE TYPE OF SECTION
C                TYPE 1 = RIGHT CIRCULAR CYLINDER  R(Z) = CONSTANT
C                TYPE 2 = CONICAL SEGMENT  R(Z) = LINEARLY VARYING FUNCTION
C                TYPE 3 = HEMI-SPHERICAL END
C                TYPE 4 = ELLIPTICAL END
C                TYPE 5 = ARBITRARY 3RD ORDER END  R(Z) = A*Z**3 + B*Z**2 + C*Z +
C AZ(I+1) = DISTANCE FROM THE BASE TO THE END OF SECTION I
      AZ(1) = 0.0
      TOTVOL = 0.0
      H = 0.0
      I = 1
```

```
 30 JOE = NTYPE(I)
    A = AAA(I)
    B = BBB(I)
    C = CCC(I)
    D = DDD(I)
    GO TO (720, 721, 722, 723, 724), JOE
720 CALL CYLIND
C SUBROUTINE CYLIND YIELDS VOLUME, MOMENT OF VOLUME, DRAFT AND VERTICAL
C           PRISMATIC COEFFICIENT OF A CIRCULAR CYLINDER
C A = RADIUS, B = C = D = 0
    GO TO 725
721 CALL CONE
C SUBROUTINE CONE YIELDS VOLUME, MOMENT OF VOLUME, DRAFT AND VERTICAL
C           PRISMATIC COEFFICIENT OF A RIGHT CIRCULAR CONE
C RADIUS = A*(ZB-AZ(I)) + B
C A = SLOPE OF RADIUS VS. Z LINE
C B = RADIUS AT AZ(I), C = D = 0
    GO TO 725
722 CALL HEMI
C SUBROUTINE HEMI YIELDS VOLUME, MOMENT OF VOLUME, DRAFT AND VERTICAL
C           PRISMATIC COEFFICIENT OF A HEMI-SPHERICAL END
C A = RADIUS, B = C = D = 0
    GO TO 725
723 CALL ELLIP
C SUBROUTINE ELLIP YIELDS VOLUME, MOMENT OF VOLUME, DRAFT AND VERTICAL
C           PRISMATIC COEFFICIENT OF A 1/2 ELLIPTICAL END
C A = MAJOR AXIS - VERTICAL, B = MINOR AXIS - HORIZONTAL, C = D = 0
    GO TO 725
724 CALL THRDOR
C SUBROUTINE THRDOR YIELDS VOLUME, MOMENT OF VOLUME, DRAFT AND VERTICAL
C           PRISMATIC COEFFICIENT OF A THIRD ORDER END
C SHAPE OF END,  R = A*Z**3 + B*Z**2 + C*Z + D    WHERE  D=0
725 IF(L - 1) 30, 726, 300
726 CALL EXIT
300 READ 301, K
301 FORMAT (I10)
C K = THE NUMBER OF DIFFERENT KH'S TO BE READ
    NOSEC = I
    READ 302, (AKH(I), I = 1,K)
302 FORMAT (8F10.4)
C AKH = KH = NON - DIMENSIONAL NUMBER K*H
    DO 13 I = 1, K
    AK(I) = AKH(I)/H
 13 CFREQ(I) = (G*AK(I))**0.5/(2.0*3.14159)
C AK(I) = AW = K = OMEGA**2/G = WAVE NUMBER
727 CONTINUE
    PRINT 7, TITLE
  7 FORMAT (1H1, 33H NEWMAN SECTION CALCULATIONS  *  10A8//)
    ZG = ZGB - H
    P1 = 0.0
    P2 = 0.0
    DO 303 I = 1, K
    Q0(I) = 0.0
303 Q1(I) = 0.0
304 I = 1
```

```
  305 IF (I - NOSEC) 306, 306, 360
  306 JOHN = NTYPE(I)
      A = AAA(I)
      B = BBB(I)
      C = CCC(I)
      D = DDD(I)
      IF (AZ(I+1) - H) 309, 309, 307
  307 PRINT 308
  308 FORMAT (42H TOUGH LUCK YOUR AZ(I+1) IS GREATER THAN H)
      CALL EXIT
  309 GO TO (730, 731, 732, 733, 734), JOHN
  730 CALL PQCYL
C SUBROUTINE PQCYL YIELDS P1, P2, Q0(N), Q1(N), (N = 1, K)       CIRCULAR
C          SECTION
      GO TO 305
  731 CALL PQCON
C SUBROUTINE PQCON YIELDS P1, P2, Q0(N), Q1(N), (N = 1, K) FOR A RIGHT

C          CIRCULAR CONICAL SECTION
      GO TO 305
  732 CALL PQHSPH
C SUBROUTINE PQHSPH YIELDS P1, P2, Q0(N), Q1(N), (N = 1, K) FOR A
C          HEMI-SPHERICAL END
      GO TO 305
  733 CALL PQELL
C SUBROUTINE PQELL YIELDS P1, P2, Q0(N), Q1(N), (N = 1, K) FOR A 1/2
C          ELLIPTICAL END
      GO TO 305
  734 CALL PQTHRD
C SUBROUTINE PQTHRD YIELDS P1, P2, Q0(N), Q1(N), (N = 1, K) FOR A THIRD
C          ORDER END
      GO TO 305
  360 CONTINUE
      READ 361, AR
  361 FORMAT (F10.4)
C AR = RADIUS OF GYRATION = (I/M)**0.5   (FT)     ACTUAL MASS
      READ 369, MOVE, MOVER
  369 FORMAT (2I10)
C IF MOVE = 1 SOLVE DAMPED EQUATIONS
C IF MOVE = 2 SOLVE UNDAMPED EQUATIONS (DAMPING IN HEAVE ONLY)
C IF MOVE = 3 SOLVE BOTH DAMPED AND UNDAMPED EQUATIONS
C IF MOVER = 0  ONLY PERFORM NEWMAN CALCULATIONS
C IF MOVER = 1  ONLY PERFORM RUDNICK CALCULATIONS
C IF MOVER = 2 (OR GREATER) PERFORM NEWMAN AND RUDNICK CALCULATIONS
      IF (MOVER - 1) 370, 32, 370
  370 CONTINUE
C     ADD IS THE RATIO BETWEEN REAL MASS AND TOTAL MASS
C     ADD = REAL MASS /(REAL MASS + ADDED MASS )
      READ 362, ADD
  362 FORMAT(F10.4)
      DO 400 I = 1, K
      AW = AK(I)
      OMEGA = (AW*G)**0.5
      QO = Q0(I)
      QR = Q1(I)
```

```
      IF (MOVF - 2) 740, 741, 742
  740 CALL EQDAMP
C SUBROUTINE EQDAMP SOLVES THE DAMPED EQUATIONS OF MOTICN (48), (49), (50) ON
C           PAGE 16 OF NEWMAN, 'THE MOTIONS OF A SPAR BUOY IN REGULAR WAVES'
      GO TO 400
  741 CALL EQUNDM
C SUBROUTINE EQUNDM SOLVES THE UNDAMPED EQUATIONS OF MOTION (34), (35), (36)
C           ON PAGE 13 OF NEWMAN, 'THE MOTIONS OF A SPAR BUOY IN REGULAR WAVES'
      GO TO 400
  742 CALL EQDAMP
      CALL EQUNDM
  400 CONTINUE
   32 CONTINUE
C THE FOLLOWING IS AN ATTEMPT TO ARRANGE A NEAT PRINT OUT OF INFORMATION
      PRINT 410, TITLE
  410 FORMAT (1H1, 27H MODEL CHARACTERISTICS   *  10A8///)
      PRINT 31, NOSEC
   31 FORMAT (23H NUMBER OF SECTIONS  = I10///)
      PRINT 411, H
  411 FORMAT (1 H DRAFT  = F10.4, 6H    FT///)
      WTON = W/2240.0
      PRINT 33, W, WTON
   330FORMAT (11H WEIGHT  = F12.4, 6H    LB5X3H = F12.4, 4X9HLONG TONS//
     1      /)
      PRINT 34, VOL
   34 FORMAT (21H DISPLACED VOLUME  = F20.4, 9H    FT**3///)
      PRINT 409, CHI
  409 FORMAT (40H VERTICAL PRISMATIC COEFFICIENT - CHI = F10.4///)
      PRINT 35, ZGB
   350FORMAT (43H HEIGHT OF CENTER OF GRAVITY ABOVE BASE  = F10.4,
     1      6H    FT,///)
      TOTMOM = 0.0
      DO 38 I = 1, NOSEC
   38 TOTMOM = TOTMOM + AMOM(I)
      BCB = TOTMOM/VOL
      PRINT 36, BCB
   360FORMAT (44H HEIGHT OF CENTER OF BUOYANCY ABOVE BASE  = F10.4,
     1      6H    FT,///)
      PRINT 416, AR
  4160FORMAT (37H RADIUS OF GYRATION (ACTUAL MASS)  = F12.8, 6H    FT
     1      ///)
      PRINT 418 , ARR
  4180FORMAT (40H RADIUS OF GYRATION (DISPLACED MASS)  = F12.8,
     1      6H    FT,///)
      PRINT 200, ARC
  2000FORMAT (45H RADIUS OF GYRATION ABOUT C (ACTUAL MASS)  = F12.8,
     1      6H    FT,///)
      PRINT 37, ROE
   37 FORMAT (21H DENSITY OF FLUID  = F10.4, 19H    LB-SEC**2/FT**4///)
      PRINT 39, G
   39 FORMAT (23H ACCELERATION OF GRAVITY  = F10.4, 13H    FT/SEC**2///)
      IF (MOVER-1) 40, 800,40
   40 CONTINUE
      PRINT 41, TITLE
   41 FORMAT (1H1, 23H NEWMAN RESONANT FREQUENCIES  *  10A8/////)
```

```
      PRINT 42
   42 FORMAT (16H HEAVE RESONANCE//)
      HEVKH = 1.0/CHI
      HEVK = HEVKH/H
      HEF = (HEVK*G)**0.5/(2.0*3.14159)
      PRINT 43, HEVKH, HEVK, HEF
   430FORMAT (7H KH   = F10.4, 10X6H K   = F10.4, 8H  FT**-1 10X
     1      6H F   = F10.4, 5H  CPS////)
      IF (P1) 47, 44, 47
   44 PRINT 45
   45 FORMAT (40H THERE IS NO RESONANCE IN PITCH OR SURGE)
      GO TO 417
   47 PRINT 48
   48 FORMAT (26H PITCH AND SURGE RESONANCE//)
      PITKH = P1*H/(P2 + AR**2 - 0.5*P1**2)
      PITK = PITKH/H
      PITF = (PITK*G)**0.5/(2.0*3.14159)
      PRINT 49, PITKH, PITK, PITF
   490 FORMAT (7H KH   = F10.4, 10X6H K   = F10.4, 8H  FT**-1 10X
     1      6H F   = F10.4, 5H  CPS)
      PRINT 401, TITLE
  401 FORMAT (1H1,30H NEWMAN MODEL P, Q VALUES  *  10A8///)
  417 PRINT 412, P1, P2
  412 FORMAT (10X6H P1 = F10.4, 11H       P2 = F10.4///)
      PRINT 413
  413 FORMAT (6X4H KH 5X3H K 3X11H FREQ (CPS) 7X7H Q0(K) 7X6H Q1(K)///)
      DO 415 I = 1, K
      PRINT 414, AKH(I), AK(I), CFREQ(I), Q0(I), Q1(I)
  414 FORMAT (F10.2, 2F10.4, 2F15.4/)
  415 CONTINUE
      GO TO (420, 430, 420), MOVE
  420 PRINT 421, TITLE
  421 FORMAT (1H1, 27H NEWMAN DAMPED MOTIONS  *  10A8///)
      PRINT 422
  4220FORMAT(7X,* KH*,7X,* K*,2X,* FREQUENCY (CPS)*   1X,* MAG ZETA1/A*,
     13X,* MAG ZETA2/A*,3X,* MAG XI/A*,2X,* PSI/KA*,5X,* PHASE LAG*///)
      DO 424 I = 1, K
      PRINT 423,AKH(I), AK(I), CFREQ(I), ZETA1(I)  ZETA2(I), XI(I),
     1PSI(I), EPSILO(I)
  423 FORMAT (F10.2, F10.4, F11.4, 5X3F13.4, F12.4, F12.4/)
  424 CONTINUE
      PRINT555
  555 FORMAT(1H1,*DIFFERENT ARRANGEMENT  OF THE RESULT FOR SPECTRAL ANAL
     1YSIS*,///)
      PRINT 666
  666 FORMAT(7X,* KH*,6X,* K*,7X,* FREQUENCY (CPS)*,5 X,* ZETA3*,13X,* X
     1I3*,13X,* PSI3*///)
  444 FORMAT(F10.2,F10.4,4F17.4/)
      DO 1000 I=1,K
 1000 PRINT444, AKH(I), AK(I), CFREQ(I), ZETA3(I), XI3(I), PSI3(I)
      IF (MOVE - 1) 440, 440, 430
  430 PRINT 431, TITLE
  431 FORMAT (1H1, 29H NEWMAN UNDAMPED MOTIONS  *  10A8//)
      PRINT 432
  4320FORMAT (7X2HKH 7X1HK  8X17HFREQUENCY (CPS)  4X10HMAG ZETA/A 6X
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```
    1      9H PAS XI/A 5X10HMAG PSI/KA///)
      PRINT 426
 4260FORMAT(5X5H 0.00 3X7H 0.0000 8X7H 0.0000 13X7H 1.0000 8X7H 1.0000
    1      8X7H-1.0000/)
      DO 434 I = 1, K
      PRINT 433, AKH(I), AK(I), CFREQ(I), UNZETA(I), UNXI(I), UNPSI(I)
 433 FORMAT (F10.2, F10.4, F15.4, 5X3F15.4/)
 434 CONTINUE
 440 CONTINUE
 500 READ 501, LPRES
 501 FORMAT (I10)
      IF (LPRES - 1) 504, 600, 600
C IF LPRES = 0 MAKE PRESSURE CALCULATIONS FOR BUOY
C IF LPRES = 1 OR LARGER DO NOT MAKE PRESSURE CALCULATIONS FOR BUOY
 504 CALL PRSCAL
C SUBROUTINE PRSCAL COMPUTES PRESSURE ON BUOY AT LOCATIONS SPECIFIED
 600 CONTINUE
      IF (MOVER - 1) 900, 800, 800
 800 CONTINUE
C STATEMENTS 800 - 900 ARE FOR RUDNICK CALCULATIONS
      BG = BCP - ZGO
      SQAR = AR**2
 850 CALL RRESON
C SUBROUTINE RRESON YIELDS RUDNICK RESONANT FREQUENCIES FOR THE BUOY
      CALL RMOT
C SUBROUTINE RMOT COMPUTES MOTIONS OF THE BUOY USING RUDNICK METHOD
 900 CONTINUE
      PRINT 855
 855 FORMAT (1H1, 50H DAMPING COEFFICIENT///)
      PRINT 856
 8560FORMAT(7X2HKH 7X1HK  8X17HFREQUENCY (CPS)  8X14HDAMPING COEFF.
    18X17HDAMP2 COEFF(ADD).//)
      DO 860 I = 1, K
      DAMP(I)=W*AK(I)*(AK(I)*G)**.5*(1.-CHI*AKH(I)*Q0(I))**2/
     1(4*ROE*G*CHI**2*H**2)
      DAMP1(I)=ADD*DAMP(I)
C     DAMP IS DAMPING COEFF. WITHOUT THE ADDED MASS
C     DAMP1 IS THE DAMPING COEFF. WITH ADDED MASS
      PRINT 857, AKH(I), AK(I), CFREQ(I), DAMP(I),DAMP1(I)
 857 FORMAT(F10.2,F10.4,F15.4,10XF15.8,10XF15.8/)
 860 CONTINUE
      END

      SUBROUTINE CYLIND
      COMMON/FIRST/A, B, C, D, I, L, SUBVOL, AREA, TOTVOL, VOL, H, CHI,
    1      W, G, ROE, CO, AZ, AMOM, HH
      DIMENSION AZ(9), AMOM(8), HH(8)
C SUBROUTINE CYLIND YIELDS VOLUME, MOMENT OF VOLUME, DRAFT AND VERTICAL
C           PRISMATIC COEFFICIENT OF A CIRCULAR CYLINDER
C A = RADIUS, B = C = D = 0
  50 AREA = 3.14159*A**2

      SUBVOL = AREA*(AZ(I+1) - AZ(I))
      TOTVOL = TOTVOL + SUBVOL
      IF (VOL - TOTVOL) 60, 52, 51
```

```
51 AMOM(I) = (0.5*(AZ(I + 1) - AZ(I)) + AZ(I))*SUBVOL
   I = I + 1
   L = 0
   RETURN
52 H = AZ(I+1)
   AMOM(I) = (0.5*(AZ(I + 1) - AZ(I)) + AZ(I))*SUBVOL
   CHI = W/(G*ROE*H*AREA)
   L = 2
   RETURN
60 TOTVOL = TOTVOL - SUBVOL
   HH(I) = (VOL - TOTVOL)/AREA
   AMOM(I) = (0.5*HH(I) + AZ(I))*3.14159*A**2*HH(I)
   H = AZ(I) + HH(I)
   CHI = W/(G*ROE*H*AREA)
   AZ(I+1) = H
   L = 2
   RETURN
   END

   SUBROUTINE CONE
   COMMON/FIRST/A, B, C, D, I, L, SUBVOL, AREA, TOTVOL, VOL, H, CHI,
  1     W, G, ROE, SO, AZ, AMOM, HH
   DIMENSION AZ(9), AMOM(8), HH(8)
C SUBROUTINE CONE YIELDS VOLUME, MOMENT OF VOLUME, DRAFT AND VERTICAL
C          PRISMATIC COEFFICIENT OF A RIGHT CIRCULAR CONE
C RADIUS = A*(Z3-AZ(I ) + B
C A = SLOPE OF RADIUS VS. Z LINE
C B = RADIUS AT AZ(I), C = D = 0
1000SUBVOL = 3.14159*(((A**2)/3.0)*(AZ(I+1)-AZ(I))**3 + A*B*
  1     (AZ(I+1)-AZ(I))**2 + (B**2)*(AZ(I+1)-AZ(I)))
   TOTVOL = TOTVOL + SUBVOL
   DIS = AZ(I+1) - AZ(I)
   IF (VOL - TOTVOL) 110, 102, 101
1010AMOM(I) = DIS*((B**2 + 2.0*B*(B + A*DIS) + 3.0*(B + A*DIS)**2)/
  1     (4.0*(B**2 + B*(B + A*DIS) + (B + A*DIS)**2)))*SUBVOL
  2     + AZ(I)*SUBVOL
   I = I + 1
   L = 0
   RETURN
102 H = AZ(I+1)
   AMOM(I) = DIS*((B**2 + 2.0*B*(B + A*DIS) + 3.0*(B + A*DIS)**2)/
  1     (4.0*(B**2 + B*(B + A*DIS) + (B + A*DIS)**2)))*SUBVOL
  2     + AZ(I)*SUBVOL
   SO = 3.14159*(A*(AZ(I+1)-AZ(I)) + B)**2
   CHI = W/(G*ROE*H*SO)
   L = 2
   RETURN
110 TOTVOL = TOTVOL - SUBVOL
   HH(I) = ((B/A**3) + 3.0*(VOL-TOTVOL)/(3.14159*A**2))**(1.0/3.0)
   DIS = HH(I) - AZ(I)
   SUBVOL = 3.14159*(((A**2)/3.0)*DIS**3 + A*B*DIS**2 + B**2*DIS)
   AMOM(I) = DIS*((B**2 + 2.0*B*(B + A*DIS) + 3.0*(B + A*DIS)**2)/
  1     (4.0*(B**2 + B*(B + A*DIS) + (B + A*DIS)**2)))*SUBVOL
  2     + AZ(I)*SUBVOL
   H = AZ(I) + HH(I)
```

```
) = 3.14159*(A*HH(I) + B)**2
HI = W/(G*ROE*H*SO)
((I+1) = H
 = 2
:TURN
ID

JBROUTINE HEMI
OMMON/FIRST/A, B, C, D, I, L, SUBVOL, AREA, TOTVOL, VOL, H, CHI,
    W, G, ROE, SO, AZ, AMOM, HH
IMENSION AZ(9), AMOM(8), HH(8)
JTINE HEMI YIELDS VOLUME, MOMENT OF VOLUME, DRAFT AND VERTICAL
    PRISMATIC COFFFICIENT OF A HEMI-SPHERICAL END
ADIUS, B = C = D = 0
JBVOL = (2.0/3.0)*3.14159*A**3
OTVOL = TOTVOL + SUBVOL
 (VOL - TOTVOL) 151, 155, 160
INT 152
ORMAT (54H      PROGRAM INADEQUATE TO FIND DEPTH OF A HEMI-SPHERE)
 = 1
TURN
 = A
MOM(I) = (5.0/8.0)*A*SUBVOL
) = 3.14159*H**2
HI = W/(G*ROE*H*SO)
 = 2
TURN
MOM(I) = (5.0/8.0)*A*SUBVOL
 = I + 1
 = 0
TURN
ID

JBROUTINE ELLIP
OMMON/FIRST/A, B, C, D, I, L, SUBVOL, AREA, TOTVOL, VOL, H, CHI,
    W, G, ROE, SO, AZ, AMOM, HH
MENSION AZ(9), AMOM(8), HH(8)
ITINE ELLIP YIELDS VOLUME, MOMENT OF VOLUME, DRAFT AND VERTICAL
    PRISMATIC COFFFICIENT OF A 1/2 ELLIPTICAL END
JOR AXIS - VERTICAL, B = MINOR AXIS - HORIZONTAL, C = D = 0
IBVOL = (2.0/3.0)*3.14159*B*A**2
TVOL = TOTVOL + SUBVOL
 (VOL - TOTVOL) 201, 205, 210
INT 202
IRMAT (5X52HPROGRAM INADEQUATE TO FIND DEPTH OF ELLIPTICAL SHAPE)
 = 1
TURN
= A/2.0
OM(I) = (5.0/8.0)*(A/2.0)*SUBVOL
 = 3.14159*(B/2.0)**2
I = W/(G*ROE*H*SO)
= 2
TURN
OM(I) = (5.0/8.0)*(A/2.0)*SUBVOL
= I + 1
```

```
      L = 0
      RETURN
      END

      SUBROUTINE THRDOR
      COMMON/FIRST/A, B, C, D, I, L, SUBVOL, AREA, TOTVOL, VOL, H, CHI,
     1     W, G, ROE, SO, AZ, AMOM, HH
      DIMENSION AZ(9), AMOM(8), HH(8)
C SUBROUTINE THRDOR YIELDS VOLUME, MOMENT OF VOLUME, DRAFT AND VERTICAL
C          PRISMATIC COEFFICIENT OF A THIRD ORDER END
C SHAPE OF END,  R = A*Z**3 + B*Z**2 + C*Z + D    WHERE  D=0
  250 Z = AZ(I+1)
      SUBVOL = 3.14159*Z**3*(C**2/3.0 + Z*(B*C + Z*((2.0*A*C + B**2)
     1     + Z*(A*B/3.0 + Z*A**2/7.0))))
      THRDM = 3.14159*Z**4*(C**2/4. + Z*(2.0*B*C/5.0 + Z*((2.0*A*C + B**2.
     1     2)/6.( + Z*(2.0*A*B/7.0 + Z*A**2/8.0))))
      TOTVOL = TOTVOL + SUBVOL
      IF (VOL - TOTVOL) 251, 255, 260
  251 PRINT 252
  252 FORMAT (5X51HPROGRAM INADEQUATE TO FIND DEPTH OF 3RD ORDER SHAPE)
      L = 1
      RETURN
  255 H = AZ (I+1)
      SO = 3.14159*(A*AZ(I+1)**3 + B*AZ(I+1)**2 + C*AZ(I+1))**2
      CHI = W/(G*ROE*H*SO)
      AMOM(I) = THRDM
      L = 2
      RETURN
  260 AMOM(I) = THRDM
      I = I + 1
      L = 0
      RETURN
      END

      SUBROUTINE PQCYL
      COMMON/FIRST/A, B, C, D, I, L, SUBVOL, AREA, TOTVOL, VOL, H, CHI,
     1     W, G, ROE, SO, AZ, AMOM, HH
      COMMON/SECOND/BB, Z1, Z0, P11, P1, ZG, P22, P2, N, K, AW, AB, AA,
     1     CA, Z, AK, Q00, Q0, Q11, Q1
      COMMON/FIFTH/TITLE, HEVKH, HEVK, HEF, BG, SQAR,
     1     MOVFR, CFREQ, PPSI, AKH
      DIMENSION AZ(9), AMOM(8), HH(8)
      DIMENSION AK(50), Q00(50), Q0(50), Q11(50), Q1(50)
      DIMENSION CFREQ(50), PPSI(50), AKH(50)
      DIMENSION TITLE (10)
C SUBROUTINE PQCYL YIELDS P1, P2, Q0(N), Q1(N), (N = 1, K) FOR A CIRCULAR
C          SECTION
  310 AREA = 3.14159*A**2
      BB = AREA*ROE*G/W
      Z1 = AZ(I+1) - H
      Z0 = AZ(I) - H
      P11 = BB*(0.5*(Z1**2 - Z0**2) - ZG*(Z1 - Z0))
      P1 = P1 + P11
      P22 = BB*((Z1**3 - Z0**3)/3.0 - (Z1**2 - Z0**2)*ZG +(Z1 - Z0)*ZG**2)
     12)
```

```
      P2 = P2 + P22
      IF(I - 1) 702, 702, 700
  700 PRINT 701
  701 FORMAT(1H1)
  702 PRINT 311, I
  311 FORMAT (29H CYLINDRICAL SECTION NUMBER  I10//)
      PRINT 315, P1, P11
  315 FORMAT (6H P1 = F10.4, 11H      P11 = F10.4)
      PRINT 316, P2, P22
  316 FORMAT (6H P2 = F10.4, 11H      P22 = F10.4///)
      DO 319 N = 1, K
      AW = AK(N)
      Q00(N) = (BB/AW)*(EXP(AW*Z1) - EXP(AW*Z0))
      Q0(N) = Q0(N) + Q00(N)
      Q11(N) = (BB/AW)*(EXP(AW*Z1)*(Z1 - 1.0/AW - ZG) - EXP(AW*ZC)*
     1      (Z0 - 1.0/AW - ZG))
      Q1(N) = Q1(N) + Q11(N)
      PRINT 314, AKH(N), AW, CFREQ(N)
  314 FORMAT (5X6H KH = F10.4, 5X5H K = F10.4, 8H  FT**-1 5X
     1      13H FREQUENCY = F10.4, 5H  CPS/)
      PRINT 317, N, Q0(N), Q00(N)
  317 FORMAT (5H N = I5,13H      Q0(N) = F10.4,14H      Q00(N) = F10.4)
      PRINT 318, N, Q1(N), Q11(N)
  318 FORMAT (5H N = I5,13H      Q1(N) = F10.4,14H      Q11(N) = F10.4//)
  319 CONTINUE
      I = I + 1
      RETURN
      END

      SUBROUTINE POCON
      COMMON/FIRST/A, B, C, D, I, L, SUBVOL, AREA, TOTVOL, VOL, H, CHI,
     1      W, G, ROE, SO, AZ, AMOM, HH
      COMMON/SECOND/BB, Z1, ZC, P11, P1, ZG, P22, P2, N, K, AW, AB, AA,
     1      CA, Z, AK, Q00, Q0, Q11, Q1
      COMMON/FIFTH/TITLE, HEVKH, HEVK, HEF, BG, SUAR,
     1      MOVER, CFREQ, RPSI, AKH
      DIMENSION AZ(9), AMOM(8), HH(8)
      DIMENSION AK(50), Q00(50), Q0(50), Q11(50), Q1(50)
      DIMENSION CFREQ(50), RPSI(50), AKH(50)
      DIMENSION TITLE (10)
C SUBROUTINE POCON YIELDS P1, P2, Q0(N), Q1(N), (N = 1, K) FOR A RIGHT
C           CIRCULAR CONICAL SECTION
  320 Z0 = AZ(I) - H
      Z1 = AZ(I+1) - H
      AB = B + A*(H - AZ(I))
      BB = 3.14159*ROE*G/W
      P11 = BB*(A**2*(Z1**4 - Z0**4)/4.0 + (2.0*A*AB - A**2*ZG)*
     1      (Z1**3 - Z0**3)/3.0 + (AB**2 - 2.0*A*AB*ZG)*(Z1**2 -
     2      Z0**2)/2.0 - AB**2*ZG*(Z1 - Z0))
      P1 = P1 + P11
      P22 = BB*(A**2*(Z1**5 - Z0**5)/5.0 + 2.0*(A*AB - A**2*ZG)*(Z1**4
     1      - Z0**4)/4.0 + (AB**2 - 4.0*A*AB*ZG + A**2*ZG**2)*
     2      (Z1**3 - Z0**3)/3.0 + 2.0*(A*AB*ZG**2 - AB**2*ZG)*(Z1**2
     3      - Z0**2)/2.0 + AB**2*ZG**2*(Z1 - Z0))
      P2 = P2 + P22
```

```
      IF(I - 1) 712,712,710
  710 PRINT 711
  711 FORMAT (1H1)
  712 PRINT 321, I
  321 FORMAT (25H CONICAL SECTION NUMBER  I10//)
      PRINT 323, P1, P11
  323 FORMAT (6H P1 = F10.4, 11H      P11 = F10.4)
      PRINT 325, P2, P22
  325 FORMAT (6H P2 = F10.4, 11H      P22 = F10.4///)
      DO 329 N = 1, K
      AW = AK(N)
      PRINT 324, AKH(N), AW, CFREQ(N)
 3240 FORMAT (5X6H KH = F10.4, 5X5H K = F10.4, 8H  FT**-1 5X
     1       13H FREQUENCY   F10.4, 5H  CPS/)
      Q00(N) = (BB/AW)*(EXP(AW*Z1)*(A**2*(Z1**2 - (2.0/AW**2)*(AW*Z1 -
     1       1.0)) + (2.0*A*AB/AW)*(AW*Z1 - 1.0) + AB**2) -
     2       EXP(AW*Z0)*(A**2*(Z0**2 - (2.0/AW**2)*(AW*Z0 - 1.0)) +
     3       (2.0*A*AB/AW)*(AW*Z0 - 1.0) + AB**2))
      Q0(N) = Q0(N) + Q00(N)
      Q11(N) = (BB/AW)*(EXP(AW*Z1)*(A**2*(Z1**3 - 3.0*Z1**2/AW +(6.0/
     1       AW**3)*(AW*Z1 - 1.0)) + (2.0*A*AB - A**2*ZG)*(Z1**2 -
     2       (2.0/AW**2)*(AW*Z1 - 1.0)) + (AB**2 - 2.0*A*AB*ZG)*
     3       (AW*Z1 - 1.0)/AW - AB**2*ZG) - EXP(AW*Z0)*(A**2*(Z0**3 -
     4       3.0*Z0**2/AW + (6.0/AW**3)*(AW*Z0 - 1.0)) + (2.0*A*AB
     5       - A**2*ZG)*(Z0**2 - (2.0/AW**2)*(AW*Z0 - 1.0)) + (AB**2
     6       - 2.0*A*AB*ZG)*(AW*Z0 - 1.0)/AW - AB**2*ZG))
      Q1(N) = Q1(N) + Q11(N)
      PRINT 326, N, Q0(N), Q00(N)
  326 FORMAT (5H N = I5,13H     Q0(N) = F10.4,14H     Q00(N) = F10.4)
      PRINT 327, N, Q1(N), Q11(N)
  327 FORMAT (5H N - I5,13H     Q1(N) = F10.4,14H     Q11(N) = F10.4//)
  329 CONTINUE
      I = I + 1
      RETURN
      END

      SUBROUTINE PQHSPH
      COMMON/FIRST/A, B, C, D, I, L, SUBVOL, AREA, TOTVOL, VOL, H, CHI,
     1       W, G, ROE, SO, AZ, AMOM, HH
      COMMON/SECOND/BB, Z1, Z0, P11, P1, ZG, P22, P2, N, K, AW, AB, AA,
     1       CA, Z, AK, Q00, Q0, Q11, Q1
      COMMON/FIFTH/TITLE, HEVKH, HEVK, HEF, BG, SQAR,
     1       MOVER, CFREQ, RPSI, AKH
      DIMENSION AZ(9), AMOM(8), HH(8)
      DIMENSION AK(50), Q00(50), Q0(50), Q11(50), Q1(50)
      DIMENSION CFREQ(50), RPSI(50), AKH(50)
      DIMENSION TITLE (10)
C SUBROUTINE PQHSPH YIELDS P1, P2, Q0(N), Q1(N), (N = 1, K) FOR A
C          HEMI-SPHERICAL END
  330 R = A
      BB = 3.14159*ROE*G/W
      AA = R - H - ZG
      P11 = BB*(2.0*AA*R**3/3.0 - R**4/4.0)
      P1 = P1 + P11
      P22 = BB*(-R**5/5.0+2.0*AA*R**4/4.0 + (R**2 - AA**2)*R**3/3.0
```

```
1        -AA*R**4 + AA**2*R**3)
      P2 = P2 + P22
      PRINT 331, I
  331 FORMAT (28H HEMI-SPHERICAL END NUMBER  I10//)
C I MUST ALWAYS = 1 IF THIS IS AN END
      PRINT 323, P1, P11
  323 FORMAT (6H P1 = F10.4, 11H     P11 = F10.4)
      PRINT 325, P2, P22
  325 FORMAT (6H P2 = F10.4, 11H     P22 = F10.4///)
      DO 339 N = 1, K
      AW = AK(N)
      PRINT 332, AKH(N), AW, CFREQ(N)
  332 FORMAT (5X6H KH = F10.4, 5X5H K = F10.4, 8H  FT**-1 5X
     1      13H FREQUENCY = F10.4, 5H  CPS/)
      Q00(N) = (BB*EXP(-AW*H)/AW)*(EXP(AW*R)*(R**2 - 2.0/AW**2) + (2.0/
     1      AW**2)*(AW*R + 1.0))
      Q0(N) = Q0(N) + Q00(N)
      Q11(N) = (BB*EXP(-AW*H)/AW)*(EXP(AW*R)*(6.0/AW**3 - 2.0*AA/AW**2
     1      - R**2/AW + AA*R**2) - R**3 - 3.0*(R**2/AW + (2.0/AW)*
     2      (AW*R - 1.0)/AW) + AA*(R**2 + 2.0*(AW*R + 1.0)/AW**2)
     3      + R**2*(AW*R + 1.0)/AW - AA*R**2)
      PRINT 333, N, Q0(N), Q00(N)
  333 FORMAT (5H N = I5,13H     Q0(N) = F10.4,14H     Q00(N) = F10.4)
      PRINT 334, N, Q1(N), Q11(N)
  334 FORMAT (5H N = I5,13H     Q1(N) = F10.4,14H     Q11(N) = F10.4//)
  339 CONTINUE
      I = I + 1
      RETURN
      END

      SUBROUTINE PQELL
      COMMON/FIRST/A, B, C, D, I, L, SUBVOL, AREA, TOTVOL, VOL, H, CHI,
     1      W, G, ROE, SO, AZ, AMOM, HH
      COMMON/SECOND/BB, Z1, Z0, P11, P1, ZG, P22, P2, N, K, AW, AB, AA,
     1      CA, Z, AK, Q00, Q0, Q11, Q1
      COMMON/FIFTH/TITLE, HEVKH, HEVK, HEF, BG, SQAR,
     1      MOVER, CFREQ, RPSI, AKH
      DIMENSION AZ(8), AMOM(8), HH(8)
      DIMENSION AK(50), Q00(50), Q0(50), Q11(50), Q1(50)
      DIMENSION CFREQ(50), RPSI(50), AKH(50)
      DIMENSION TITLE (10)
C SUBROUTINE PQELL YIELDS P1, P2, Q0(N), Q1(N), (N = 1, K) FOR A 1/2
C           ELLIPTICAL END
  340 BB = 3.14159*ROE*G/W
      AA = H - A/2.0
      CA = (B/A)**2
      P11 = BB*CA*((H**4 - AA)/4.0 - (2.0*(AA - ZG/2.0)/3.0)*(H**3 - AA
     1      **3) + (H**2 - H*A - 2.0*AA*ZG)*((H**2 - AA)/2.0)
     2      - (H**2 - H*A)*ZG*A/2.0)
      P1 = P1 + P11
      PRINT 341
  341 FORMAT (24H ELLIPTICAL END NUMBER  I10//)
C I MUST ALWAYS = 1 IF THIS IS AN END
      P22 = BB*CA*((AA**5 - H**5)/5.0 + AA*(H**4 - AA**4)/2.0 +
     1      (H**2 - H*A)*(AA**3 - H**3)/3.0 + 2.0*ZG*(((AA**4 - H**4)/4.0
```

```
    2         2.0*AA*(H**3 - AA**3)/3.0 + (H**2 - H*A)*(AA**2
    3         - H**2)/2.0) + ZG**2*(H**2 - H*A)*(A/2.0)))
      P2 = P2 + P22
      PRINT 323, P1, P11
  323 FORMAT (6H P1 = F10.4, 11H        P11 = F10.4)
      PRINT 325, P2, P22
  325 FORMAT (6H P2 = F10.4, 11H        P22 = F10.4///)
      DO 349 N = 1, K
      AW = AK(N)
      PRINT 342, AKH(N), AW, CFREQ(N)
  342 FORMAT (5X6H KH = F10.4, 5X5H K = F10.4, 8H  FT**-1 5X
    1         13H FREQUENCY = F10.4, 5H  CPS/)
      Q00(N) = (BB*CA/AW)*(EXP(AW*(-H + A/2.0))*(-AA**2 + (2.0/AW**2)*
    1         (AW*AA + 1.0)*(AA - 1.0) - H**2 + H*A) + EXP(-H*AK)*
    2         (2.0*H**2 + (2.0/AW**2)*(-AW*H - 1.0)*(AA - 1.0) - H*A))
      Q0(N) = Q0(N) + Q00(N)
      Q11(N) = (BB*CA/AW)*(EXP(-AW*AA)*(AA**3 + (3.0/AW)*AA**3 +
    1         (6.0/AW**3)*(AW*AA + 1.0) - 2.0*AA - ZG*(AA**2 +
    2         (2.0/AW**2)*(AW*AA + 1.0)) + (H**2 - H*A - 2.0*ZG*
    3         AA)*(AW*AA - 1.0)/AW + ZG*(H**2 - H*A)) + EXP(-AW*H)*
    4         (-H**3 - 3.0*H**2/AW - 6.0*(AW*H + 1.0)/AW**3 + 2.0*
    5         (AA - ZG)*(H**2 + 2.0*(AW*H + 1.0)/AW**2) - (H**2 -
    6         H*A - 2.0*ZG*AA)*(AW*H + 1.0)/AW - ZG*(H**2 - H*A)))
      Q1(N) = Q1(N) + Q11(N)
      PRINT 343, N, Q0(N), Q00(N)
  343 FORMAT (5H N = I5,13H      Q0(N) = F10.4, 14H      Q00(N) = F10.4)
      PRINT 344, N, Q1(N), Q11(N)
  344 FORMAT (5H N = I5,13H      Q1(N) = F10.4,14H      Q11(N) = F10.4//)
  349 CONTINUE
      I = I + 1
      RETURN
      END

      SUBROUTINE POTHRD
      COMMON/FIRST/A, B, C, D, I, L, SUBVOL, AREA, TOTVOL, VOL, H, CHI,
     1       W, G, ROE, SO, AZ, AMOM, HH
      COMMON/SECOND/BB, Z1, ZO, P11, P1, ZG, P22, P2, N, K, AW, AB, AA,
     1       CA, Z, AK, Q00, Q0, Q11, Q1
      COMMON/FIFTH/TITLE, HEVKH, HEVK, HEF, BG, SQAR,
     1       MOVER, CFREQ, RPSI, AKH
      DIMENSION AZ(9), AMOM(8), HH(8)
      DIMENSION AK(50), Q00(50), Q0(50), Q11(50), Q1(50)
      DIMENSION CFREQ(50), RPSI(50), AKH(50)
      DIMENSION TITLE (12)
C SUBROUTINE POTHRD YIELDS P1, P2, Q0(N), Q1(N), (N = 1, K) FOR A THIRD
C            ORDER END
  350 AA = ZG + H
      Z = AZ(I+1)
      BB = 3.14159*ROE*G/W
      P11 = BB*Z**3*(Z*(C**2/4.0 + Z*(2.0*B*C/5.0 + Z*((2.0*A*C + B**2)
    1         /6.0 + Z*(2.0*A*B/7.0 + Z*A**2/8.0)))) - AA*(C**2/3.0
    2         + Z*(B*C/2.0 + Z*((2.0*A*C + B**2)/5.0 + Z*(A*B/3.0 +
    3         Z*A**2/7.0)))))
      P1 = P1 + P11
      P22 = BB*Z**3*(Z**2*(C**2/5.0 + Z*(B*C/3.0 + Z*((2.0*A*C + B**2)
```

```
1      /7.0 + Z*(A*B/4.0 + Z*A**2/9.0)))) - 2.0* A*Z*(C**2/4.0
2      + Z*(2.0*B*C/5.0 + Z*((2.0*A*C + B**2)/6.0 + Z*(2.0*
3      A*B/7.0 + Z*A**2/8.0)))) + AA**2*(C**2 + Z*(B*C/2.0
4      + Z*((2.0*A*C + B**2)/5.0 + Z*(A*B/3.0 + Z*A**2/7.0)))))
      P2 = P2 + P22
      PRINT 323, P1, P11
  323 FORMAT (6H P1 = F10.4, 11H     P11 = F10.4)
      PRINT 325, P2, P22
  325 FORMAT (6H P2 = F10.4, 11H     P22 = F10.4///)
      Z = -H + AZ(I+1)
      DO 359 N = 1, K
      AW = AK(N)
      Z1 = EXP(AW*Z)*(AW*Z - 1.0)/AW**2
      Z2 = (Z**2*EXP(AW*Z) - 2.0*Z1)/AW
      Z3 = (Z**3*EXP(AW*Z) - 3.0*Z2)/AW
      Z4 = (Z**4*EXP(AW*Z) - 4.0*Z3)/AW
      Z5 = (Z**5*EXP(AW*Z) - 5.0*Z4)/AW
      Z6 = (Z**6*EXP(AW*Z) - 6.0*Z5)/AW
      Z7 = (Z**7*EXP(AW*Z) - 7.0*Z6)/AW
      PRINT 352, AKH(N), AW, CFREQ(N)
  352 FORMAT (5X6H KH = F10.4, 5X5H K = F10.4, 8H  FT**-1 5X
    1      13H FREQUENCY = F10.4, 5H  CPS/)
      Q00(N) = BB*(A**2*Z6 + 2.0*A*B*Z5 + (2.0*A*C + B**2)*Z4
    1      + 2.0*B*C*Z3 + C**2*Z2)
      Q0(N) = Q0(N) + Q00(N)
      Q11(N) = BB*(A**2*Z7 + 2.0*A*B*Z6 + (2.0*A*C + B**2)*Z5 + 2.0*
    1      B*C*Z4 + C**2*Z2 - ZG*(A**2*Z6 + 2.0*A*B*Z5 +
    2      (2.0*A*C + B**2)*Z4 + 2.0*B*C*Z3 + C**2*Z2))
      Q1(N) = Q1(N) + Q11(N)
      PRINT 353, N, Q0(N), Q00(N)
  353 FORMAT (5H N = I5, 13H     Q0(N) = F10.4,14H     Q00(N) = F10.4)
      PRINT 354, N, Q1(N), Q11(N)
  354 FORMAT (5H N = I5,13H     Q1(N) = F10.4,14H     Q11(N) = F10.4//)
  359 CONTINUE
      I = I + 1
      RETURN
      END

      SUBROUTINE EQDAMP
      COMMON/FIRST/A, B, C, D, I, L, SUBVOL, AREA, TOTVOL, VOL, H, CHI,
    1      W, G, ROF, SO, AZ, AMOM, HH, ADD
      COMMON/SECOND/BB, Z1, Z0, P11, P1, ZG, P22, P2, N, K, AW, AB, AA,
    1      CA, Z, AK, Q00, Q0, Q11, Q1
      COMMON/THIRD/QQ, QR, OMEGA, AR, ZETA, PSI, XI, EPSILO,
    1      UNZETA, UNPSI, UNXI, ZETA1, ZETA2, ZETA3, ZETA4, XI3, PSI3
      DIMENSION AZ(9), AMOM(8), HH(8)
      DIMENSION AK(50), Q00(50), Q0(50), Q11(50), Q1(50)
      DIMENSION ZETA(50), PSI(50), XI(50), EPSILO(50), UNZETA(50),
    1      UNPSI(50), UNXI(50)
      DIMENSION ZETA1(50),ZETA2(50),ZETA3(50),ZETA4(50)
      DIMENSION PSI3(50), XI3(50)
C SUBROUTINE EQDAMP SOLVES THE DAMPED EQUATIONS OF MOTION (48), (49), (50) ON
C          PAGE 16 OF NEWMAN, 'THE MOTIONS OF A SPAR BUOY IN REGULAR WAVES'
      U1 = 1.0-CHI*AW*H
      U2 = ADD-CHI*AW*H
```

```
      T = 1.0 - CHI*AW*H*QQ
      S = 0.5*W*AW**3/(G*ROE*OMEGA)
      TT=W*AW**2*T**2/(2.*G*ROE*CHI*H)
      R1 = S*(-2.0*P1*QQ*QR + 2.0*QR**2 + P2*QQ**2 - QQ**2*P1/AW +
     1     QQ**2*AR**2)
      R2 = P1**2 - 2.0*(P2 - P1/AW + AR**2)
      R3 = 2.0*(2.0*QR - P1*QQ)
      W1 = S*(2.0*QQ*QR*P1 - AR**2*QQ**2 + P1*QQ**2/AW - P2*QQ**2 - 2.0
     1     *QR**2)
      W2 = 2.0*(P2 - P1/AW + AR**2) - P1**2
      W3 = 2.0*(QR*P1 - QQ*(AR**2 + P2 - P1/AW))
C     ZETA1 IS THE SOLUTION OF THE EQ. WITHOUT ADDED MASS
C     ZETA2(I) IS THE SOLUTION OF THE EQUATION WITH ADDED MASS
      ZETA1(I)=T/(U1**2+TT**2)**.5
      ZETA2(I)=ADD*T/(U2**2+(ADD*TT)**2)**.5
C PSI = MAGNITUDE OF PITCH AMPLITUDE/WAVE SLOPE
      PSI(I) = R3/(AW*(R2**2 + R1**2*OMEGA**2)**0.5)
C XI = MAGNITUDE OF SURGE AMPLITUDE/WAVE AMPLITUDE
      XI(I) =-W3/(W2**2 + W1**2*OMEGA**2)**0.5
      XI(I) = ABS(XI(I))
      ZETA3(I)=(ZETA2(I)*OMEGA**2)**2
      XI3(I)=(XI(I)*OMEGA**2)**2
      PSI3(I)=(PSI(I)*AW)**2
      EPSILO(I)=AW
      RETURN
      END

      SUBROUTINE EQUNDM
      COMMON/FIRST/A, B, C, D, I, L, SUBVOL, AREA, TOTVOL, VOL, H, CHI,
     1     W, C, ROE, SO, AZ, AMOM, HH
      COMMON/SECOND/BB, Z1, Z0, P11, P1, ZG, P22, P2, N, K, AW, AB, AA,
     1     CA, Z, AK, Q00, Q0, Q11, Q1
       COMMON/THIRD/QQ, QR, OMEGA, AR, ZETA, PSI, XI, EPSILO,
     1     UNZETA, UNPSI, UNXI
      DIMENSION AZ(8), AMOM(8), HH(8)
      DIMENSION AK(50), Q00(50), Q0(50), Q11(50), Q1(50)
      DIMENSION ZETA(50), PSI(50), XI(50), EPSILO(50), UNZETA(50),
     1     UNPSI(50), UNXI(50)
C SUBROUTINE EQUNDM SOLVES THE UNDAMPED EQUATIONS OF MOTION (34), (35), (36)
C          ON PAGE 13 OF NEWMAN, 'THE MOTIONS OF A SPAR BUOY IN REGULAR WAVES'
  385 UNZETA(I) = (1.0 - CHI*QQ*AW*H)/(1.0 - CHI*AW*H)
C UNZETA = UNDAMPED MAGNITUDE OF HEAVE AMPLITUDE/WAVE AMPLITUDE  FORMULA (34)
      UNPSI(I) =-2.0*(P1*QQ - 2.0*QR)/(AW*(2.0*(P2 + AR**2 - P1/AW) -
     1     P1**2))
C UNPSI = UNDAMPED MAGNITUDE OF PITCH AMPLITUDE/WAVE SLOPE  FORMULA (36)
      UNXI(I) =-2.0*(P1*QR - QQ*(P2 + AR**2 - P1/AW))/(2.0*(P2 + AR**2
     1     - P1/AW) - P1**2)
      UNXI(I) = ABS(UNXI(I))
C UNXI = UNDAMPED MAGNITUDE OF SURGE AMPLITUDE/WAVE AMPLITUDE  FORMULA (35)
      RETURN
      END

      SUBROUTINE PRECAL
      DIMENSION Z(5), BKR(80), PHE(80), P(80), THETA(5)
      G=32.14
```

```
      PI=3.14159
      READ10, R,DEL
   10 FORMAT(2F10.4)
      READ1,L,M,N
    1 FORMAT(3I10)
      READ2,(THETA(I),I=1,L)
    2 FORMAT(8F10.5)
      READ2,(Z(J),J=1,M)
      DO3 I=1,L
      PRINT77
   77 FORMAT(1H1)
      PRINT54,THETA(I)
   54 FORMAT(* THE ANGLE OF THE ORIENTATION = *,F7.5,* RADIANS*/)
      DO4 J=1,M
      PRINT55, Z(J)
   550FORMAT(//5X,* THE DISTANCE FROM THE FREE SURFACE = *,F10.5,* FEET*
     1)
      DO5 K=1,N
      BKR(K)=DEL*(FLOAT(K))
      ALPA=COS(THETA(I))
      BAI=2.*BKR(K)*ALPA
      BUNE=SQRT(1.+BAI**2)
      AM=BKR(K)*Z(J)/R
      P(K)=EXP(AM)*BUNE
      PHE(K)=ATAN(BAI)
C     P IS NON-DIMENSIONALIZED DYNAMIC PRESSURE=P/RHO*G*A
    5 CONTINUE
      PRINT 1000
 1000 FORMAT(//15X,*NO.*,17X,*KR*,12X,*P/RHO.G.A*,9X,*PHASE ANGLE*/)
      DO100 K=1,N
  100 PRINT99, K, BKR(K), P(K), PHE(K)
   99 FORMAT(8X,I10,3F20.4/)
    4 CONTINUE
    3 CONTINUE
      RETURN
      END

      SUBROUTINE RRESON
      COMMON/FIRST/A, B, C, D, I, L, SUBVOL, AREA, TOTVOL, VOL, H, CHI,
     1     W, G, ROE, SO, AZ, AMOM, HH
      COMMON/SECOND/BB, Z1, Z0, P11, P1, ZG, P22, P2, N, K, AW, AB, AA,
     1     CA, Z, AK, Q00, Q0, Q11, Q1
      COMMON/FIFTH/TITLE, HEVKH, HEVK, HEF, BG, SQAR,
     1     MOVER, CFREQ, RPSI, AKH
      DIMENSION AZ(?), AMOM(8), HH(8)
      DIMENSION CFREQ(50), RPSI(50), AKH(50)
      DIMENSION AK(50), Q00(50), Q0(50), Q11(50), Q1(50)
      DIMENSION TITLE (10)
C SUBROUTINE RRESON YIELDS RUDNICK RESONANT FREQUENCIES FOR THE BUOY
      PRINT 1, TITLE
    1 FORMAT (1H1, 34H PUDNICK RESONANT FREQUENCIES  *  10A8/////)
      PRINT 3
    3 FORMAT (16H HEAVE RESONANCE//)
      PRINT 7, HEVKH, HEVK, HEF
   70FORMAT (7H KH   = F10.4, 10X6H K   = F10.4, 8H  FT**-1 10X6H F  =
```

```
  1      F10.4, 5H  CPS////)
      PRINT 10
   10 FORMAT (16H PITCH RESONANCE//)
      PITK = P1/(P2 + SQAR)
      PITKH = PITK*H
      PITF = (PITK*G)**0.5/(2.0*3.14159)
      PRINT 15, PITKH, PITK, PITF
  150FORMAT (7H KH  = F10.4, 10X6H K  = F10.4, 8H  FT**-1 10X6H F  =
  1      F10.4, 5H  CPS////)
      PRINT 20
   20 FORMAT (28H THERE IS NO SURGE RESONANCE)
      RETURN
      END

      SUBROUTINE RMOT
      COMMON/FIRST/A, B, C, D, I, L, SURVOL, AREA, TOTVOL, VOL, H, CHI,
     1      W, G, ROE, SO, AZ, AMOM, HH
      COMMON/SECOND/BB, Z1, Z0, P11, P1, ZG, P22, P2, N, K, AW, AB, AA,
     1      CA, Z, AK, Q00, Q0, Q11, Q1
      COMMON/THIRD/QQ, QR, OMEGA, AR, ZETA, PSI, XI, EPSILO,
     1      UNZETA, UNPSI, UNXI
      COMMON/FIFTH/TITLE, HEVKH, HEVK, HEF, BG, SQAR,
     1      MOVER, CFREQ, RPSI, AKH
      DIMENSION AZ(8), AMOM(8), HH(8)
      DIMENSION AK(50), Q00(50), Q0(50), Q11(50), Q1(50)
      DIMENSION ZETA(50), PSI(50), XI(50), EPSILO(50), UNZETA(50),
     1      UNPSI(50), UNXI(50)
      DIMENSION CFREQ(50), RPSI(50), AKH(50)
      DIMENSION TITLE (10)
C SUBROUTINE RMOT COMPUTES MOTIONS OF THE BUOY USING RUDNICK METHOD
      PRINT 1, TITLE
    1 FORMAT (1H1, 21H RUDNICK MOTIONS  *  10A8///)
      PRINT 3
   30FORMAT (7X3HKH 7X1HK  8X17HFREQUENCY (CPS)  4X10HMAG ZETA/A 6X
     1      9H MAG XI/A 7X 10HMAG PSI/KA///)
      PRINT 5
   50FORMAT (5X5H 0.00 3X7H 0.0000 8X 7H 0.0000 13X7H 1.0000 8X
     1      7H 1.0000 8X7H-1.0000/)
      IF (MOVER - 1) 7, 15, 15
    7 PRINT 8
    8 FORMAT (35H EXIT RMOT STEP 8 MOVER LESS THAN 1)
      CALL EXIT
   15 CONTINUE
      DO 25 I = 1, K
      RPSI(I) = (2.0*Q1(I) - P1*Q0(I))/(AK(I)*(P2 + SQAR- P1/AK(I)))
      PRINT 18, AKH(I), AK(I), CFREQ(I), UNZETA(I), Q0(I), RPSI(I)
   18 FORMAT (F10.2, F10.4, F15.4, 5X3F15.4/)
   25 CONTINUE
      RETURN
      END
```

```
 5     AKH(I)
       WAVE NUMBER TIMES DRAFT, KH (FT)
       FORMAT(8F10.4)
       8/K CARDS - 8 KH,S PER CARD

 6     AR
       RADIUS OF GYRATION OF ACTUAL MASS (FT)
       FORMAT(2F10.4)
       1 CARD
 7     MOVE,MOVER
       NUMBERS TELLING MACHINE WHICH COMPUTATIONS TO PERFORM. (SEE NO.
       369 )
       FORMAT(2I10)
       1 CARD

 8     ADD
       THE ADDED MASS COEFFICIENT = THE RATIO OF REAL MASS AND TOTAL
       MASS
       1 CARD

 9     LPRES
       NUMBER TELLING MACHINE WHETHER TO COMPUTE PRESSURES
       SEE NO. 500
       FORMAT(I10)
       1 CARD

*** FOR PRESSURE CALCULATION ***

10     R,DEL
       MAX RADIUS OF MODEL(FT), INCREMENT OF KR VALUE
       FORMAT(2F10.4)
       1 CARD

11     L,M,N
       NUMBER OF ORIENTATIONS,NUMBER OF DEPTHS, NUMBER OF KR VALUES
       FORMAT(3I10)
       1 CARD

12     THETA(I) (I=1,L)
       ANGLE OF ORIENTATIONS (RADIANS )
       FORMAT(3F10.5)
       L/8 CARDS

13     Z(J) (J=1,M)
       DISTANCES FROM FREE SURFACE (FT)
       FORMAT(8F10.5)
       M/8 CARDS
* THIS IS THE LAST DATA CARD
```

APPENDIX B --- A SAMPLE PROBLEM ---

```
C      SAMPLE PROBLEM INPUT DATA

MODEL 4 CONE BOTTOM (2 ND EXP.)
         2      32.15    1.9367     11.78       .791
         1         2       .25        .75
         2         1        2.       .1875
         5
.5         1.0         1.5         2.0        2.5
      .642
         3           3
     .9428

  .1875     0.1
         3           2           10
 .0        1.5708     3.14159
  .125    -1.4583
```

```
CONIC L SECTION NUMBER          1

P1 =      -. 253     P11 =     -.1293
P2 =       . 170     P22 =      .0178

     KM =     .5000      K =     .2660  FT**-1     FREQUENCY =      .4054  CPS

N =     1     W0(N) =      .0310     W00(N) =      .0310
N =     1     W1(N) =     -.0187     W11(N) =     -.0187

     KM =    1.0000      K =     .5320  FT**-1     FREQUENCY =      .6582  CPS

N =     2     W0(N) =      .0198     W00(N) =      .0198
N =     2     W1(N) =     -.0119     W11(N) =     -.0119

     KM =    1.5000      K =     .7980  FT**-1     FREQUENCY =      .8062  CPS

N =     3     W0(N) =      .0126     W00(N) =      .0126
N =     3     W1(N) =     -.0076     W11(N) =     -.0076

     KM =    2.0000      K =    1.0640  FT**-1     FREQUENCY =      .9309  CPS

N =     4     W0(N) =      .0080     W00(N) =      .0080
N =     4     W1(N) =     -.0048     W11(N) =     -.0048

     KM =    2.5000      K =    1.3300  FT**-1     FREQUENCY =     1.0407  CPS

N =     5     W0(N) =      .0051     W00(N) =      .0051
N =     5     W1(N) =     -.0031     W11(N) =     -.0031
```

```
CYLINDRICAL SECTION NUMBER      2

F1  =      .[illegible]      F11 =      .2614
F2  =      .[illegible]      F22 =      .2844

       KM =     .5000     K =     .2600  FT**-1     FREQUENCY =     .4654  CPS
N  =    1     W0(N) =    .8030    W00(N) =    .7760
N  =    1     W1(N) =    .2388    W11(N) =    .2575

       KM =    1.0000     K =     .5320  FT**-1     FREQUENCY =     .6582  CPS
N  =    2     W0(N) =    .6560    W00(N) =    .6362
N  =    2     W1(N) =    .2369    W11(N) =    .2488

       KM =    1.5000     K =     .7980  FT**-1     FREQUENCY =     .8062  CPS
N  =    3     W0(N) =    .5449    W00(N) =    .5323
N  =    3     W1(N) =    .2302    W11(N) =    .2377

       KM =    2.0000     K =    1.0640  FT**-1     FREQUENCY =     .9309  CPS
N  =    4     W0(N) =    .4598    W00(N) =    .4518
N  =    4     W1(N) =    .2207    W11(N) =    .2256

       KM =    2.5000     K =    1.3300  FT**-1     FREQUENCY =    1.0407  CPS
N  =    5     W0(N) =    .3938    W00(N) =    .3867
N  =    5     W1(N) =    .2101    W11(N) =    .2132
```

MODEL CHARACTERISTICS * MODEL A CONE BOTTOM (2 ND EXP.)

NUMBER OF SECTIONS = 2

DRAFT = 1.8796 FT

WEIGHT = 11.7800 LB = .0053 LONG TONS

DISPLACED VOLUME = .1892 FT**3

VERTICAL PRISMATIC COEFFICIENT = CMI = .9113

HEIGHT OF CENTER OF GRAVITY ABOVE BASE = .7900 FT

HEIGHT OF CENTER OF BUOYANCY ABOVE BASE = 1.0221 FT

RADIUS OF GYRATION (ACTUAL MASS) = .64200000 FT

RADIUS OF GYRATION (DISPLACED MASS) = I FT

RADIUS OF GYRATION ABOUT C (ACTUAL MASS) = I FT

DENSITY OF FLUID = 1.9367 LB-SEC**2/FT**4

ACCELERATION OF GRAVITY = 32.1500 FT/SEC**2

NEWMAN RESONANT FREQUENCIES * MODEL A CONE BOTTOM (4 NO EXP.)

HEAVE RESONANCE

KH = 1.0973 K = .5830 FT**-1 F = .6895 CPS

PITCH AND SURGE RESONANCE

KH = .6306 K = .3387 FT**-1 F = .5652 CPS

NEW41. MODEL P, Q VALUES * MODEL A CONE BOTTOM (2 NO EXP.)

P1 = .2321 P2 = .3002

KM	K	FREQ (CPS)	Q0(K)	Q1(K)
.50	.2660	.4054	.8030	.2388
1.00	.5320	.6582	.6560	.2309
1.50	.7980	.8002	.5449	.2302
2.00	1.0640	.9309	.4598	.2207
2.50	1.3300	1.0407	.3938	.2101

NEWMAN DAMPED MOTIONS * MODEL A CONE BOTTOM (2 ND EXP.) DEC 2,68

KH	K	FREQUENCY (CPS)	MAG ZETA1/A	MAG ZETA2/A	MAG XI/A	PSI/KA	PHASE LAG
.50	.2660	.4654	1.1649	1.6272	.9834	5.0446	.2660
1.00	.5320	.6582	4.5339	12.0147	.5001	2.4270	.5320
1.50	.7980	.8062	.6952	.5671	.4467	1.0603	.7980
2.00	1.0640	.9309	.1968	.1735	.3767	.6732	1.0640
2.50	1.3300	1.0407	.0804	.0725	.3191	.4836	1.3300

DIFFERENT ARRANGEMENT OF THE RESULT FOR SPECTRAL ANALYSIS

KH	K	FREQUENCY (CPS)	ZETA3	XI3	PSI3
.50	.2660	.4654	110.1560	70.7385	2.4171
1.00	.5320	.6582	42231.7421	74.9373	1.0672
1.50	.7980	.8062	211.0832	131.3272	.7160
2.00	1.0640	.9309	35.2231	166.0275	.5131
2.50	1.3300	1.0407	9.0234	186.2079	.4138

NEWMAN UNDAMPED MOTIONS * MODEL A CONE BOTTOM (2 ND EXP.) DEC.2?(

KH	K	FREQUENCY (CPS)	MAG ZETA/A	MAG XI/A	MAG PSI/KA
0.00	0.0000	0.0000	1.0000	1.0000	-1.0000
.50	.2660	.4654	1.1649	.9834	-5.8446
1.00	.5320	.6582	4.5358	.5061	3.4270
1.50	.7980	.8062	-.6952	.4467	1.0604
2.00	1.0640	.9309	-.1968	.3767	.6733
2.50	1.3300	1.0407	-.0804	.3191	.4838

THE ANGLE OF THE ORIENTATION = 0. RADIANS

THE DISTANCE FROM THE FREE SURFACE = -.12500 FEET

NO.	KR	P/RHO.G.A	PHASE ANGLE
1	.1000	.9540	.1974
2	.2000	.9426	.3805
3	.3000	.9548	.5404
4	.4000	.9609	.6747
5	.5000	1.0133	.7854
6	.6000	1.0471	.8761
7	.7000	1.0789	.9505
8	.8000	1.1069	1.0122
9	.9000	1.1301	1.0637
10	1.0000	1.1480	1.1071

THE DISTANCE FROM THE FREE SURFACE = -1.45830 FEET

NO.	KR	P/RHO.G.A	PHASE ANGLE
1	.1000	.4685	.1974
2	.2000	.2273	.3805
3	.3000	.1131	.5404
4	.4000	.0571	.6747
5	.5000	.0289	.7854
6	.6000	.0147	.8761
7	.7000	.0074	.9505
8	.8000	.0037	1.0122
9	.9000	.0019	1.0637
10	1.0000	.0009	1.1071

THE ANGLE OF THE ORIENTATION = 1.57080 RADIANS

THE DISTANCE FROM THE FREE SURFACE = -.12500 FEET

NO.	KR	P/RHO.G.A	PHASE ANGLE
1	.1000	.9355	-.0000
2	.2000	.8752	-.0000
3	.3000	.8187	-.0000
4	.4000	.7659	-.0000
5	.5000	.7165	-.0000
6	.6000	.6703	-.0000
7	.7000	.6271	-.0000
8	.8000	.5866	-.0000
9	.9000	.5488	-.0000
10	1.0000	.5134	-.0000

THE DISTANCE FROM THE FREE SURFACE = -1.45830 FEET

NO.	KR	P/RHO.G.A	PHASE ANGLE
1	.1000	.4594	-.0000
2	.2000	.2111	-.0000
3	.3000	.0970	-.0000
4	.4000	.0446	-.0000
5	.5000	.0205	-.0000
6	.6000	.0094	-.0000
7	.7000	.0043	-.0000
8	.8000	.0020	-.0000
9	.9000	.0009	-.0000
10	1.0000	.0004	-.0000

THE ANGLE OF THE ORIENTATION = 3.14159 RADIANS

THE DISTANCE FROM THE FREE SURFACE = -.12500 FEET

NO.	KR	P/RHO.G.A	PHASE ANGLE
1	.1000	.9540	-.1974
2	.2000	.9426	-.3805
3	.3000	.9548	-.5404
4	.4000	.9809	-.6747
5	.5000	1.0133	-.7854
6	.6000	1.0471	-.8761
7	.7000	1.0789	-.9505
8	.8000	1.1069	-1.0122
9	.9000	1.1301	-1.0637
10	1.0000	1.1480	-1.1071

THE DISTANCE FROM THE FREE SURFACE = -1.45830 FEET

NO.	KR	P/RHO.G.A	PHASE ANGLE
1	.1000	.4085	-.1974
2	.2000	.2273	-.3805
3	.3000	.1131	-.5404
4	.4000	.0571	-.6747
5	.5000	.0289	-.7854
6	.6000	.0147	-.8761
7	.7000	.0074	-.9505
8	.8000	.0037	-1.0122
9	.9000	.0019	-1.0637
10	1.0000	.0009	-1.1071

RUDNICK RESONANT FREQUENCIES * MODEL A CONE BOTTOM (2 ND EXP.)

HEAVE RESONANCE

KH = 1.0973 K = .5830 FT**-1 F = .6095 CPS

PITCH RESONANCE

KH = .6165 K = .3259 FT**-1 F = .5152 CPS

THERE IS NO SURGE RESONANCE

KH	K	FREQUENCY (CPS)	MAG ZETA/A	MAG XI/A	MAG PSI/KA
0.00	0.0000	0.0000	1.0000	1.0000	-1.0000
.50	.2660	.4654	1.1649	.8030	-6.8267
1.00	.5320	.6582	4.5358	.6560	5.1901
1.50	.7980	.8062	-.6952	.5449	.9926
2.00	1.0640	.9309	-.1968	.4598	.6366
2.50	1.3300	1.0407	-.0804	.3938	.4596

DAMPING COEFFICIENT

KM	K	FREQUENCY (CPS)	DAMPING COEFF.	DAMP2 COEFF(ADD).
.50	.2660	.4654	.00504193	.00475354
1.00	.5320	.6582	.00573686	.00540871
1.50	.7980	.8062	.00464193	.00399919
2.00	1.0640	.9309	.00262972	.00247930
2.50	1.3300	1.0407	.00148057	.00139589

Adee, Bruce H.
Bai, Kwang June

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College of Engineering, University of California Berkeley, Report NA 70-4, July 1970. v + 84 pp.

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Key words: Spar, Spar Buoy, Spar Platform, Stable Platform, Platform Motions

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Adee, Bruce H.
Bai, Kwang June

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